Optimal Finite Blocklength Code Design on the Binary Erasure Channel

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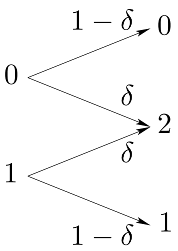
1 Introduction

In traditional coding theory, it is the goal to find good codes that operate close to the ultimate limit of the **channel capacity** as introduced by Shannon. In this work we would like to break away from these traditional simplifications and instead focus on an optimal (i.e., **minimum error probability**) design of codes for a certain given channel and a given fixed blocklength. Since for very short blocklength, it is not realistic to transmit large quantities of information, we start by looking at codes with only a few codewords, so called **ultra-small block-codes**. We introduce a new class of codes, called **fair weak flip codes** (the number of zeros and ones are almost equal under certain conditions on the blocklength n) and prove that they have beautiful **quasi-linear** properties.

2 Channel Models

We consider the binary erasure channel (BEC) with erasure probability δ:



3 Optimal Code Design

Define the following fair weak flip codes:

$$\mathscr{C}_{\text{BEC}}^{(5,10)*} \triangleq \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1&1&1\\ 0&1&1&1&0&0&0&1&1&1\\ 1&0&1&1&0&1&1&0&0&1\\ 1&1&0&1&1&0&1&0&1&0 \end{pmatrix}$$

$$\mathscr{C}_{\text{BEC}}^{(6,10)*} \triangleq \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&1&1&1\\ 0&1&1&1&0&0&0&1&1&1\\ 1&0&1&1&0&1&1&0&0&1\\ 1&1&0&1&1&0&1&0&1&0\\ 1&1&1&0&1&1&0&1&0&0 \end{pmatrix}$$

Theorem 1. *For a BEC and for any n being a multiple of 10, an* **optimal codebook** *with $\mathrm{M} = 5$ or $\mathrm{M} = 6$ codewords is*

$$\left(\mathscr{C}_{\text{BEC}}^{(\mathrm{M},10)*}\ \mathscr{C}_{\text{BEC}}^{(\mathrm{M},10)*} \cdots \mathscr{C}_{\text{BEC}}^{(\mathrm{M},10)*}\right)_{\mathrm{M}\times(n \bmod 10 = 0)}$$

4 Optimal Exact Error Probability

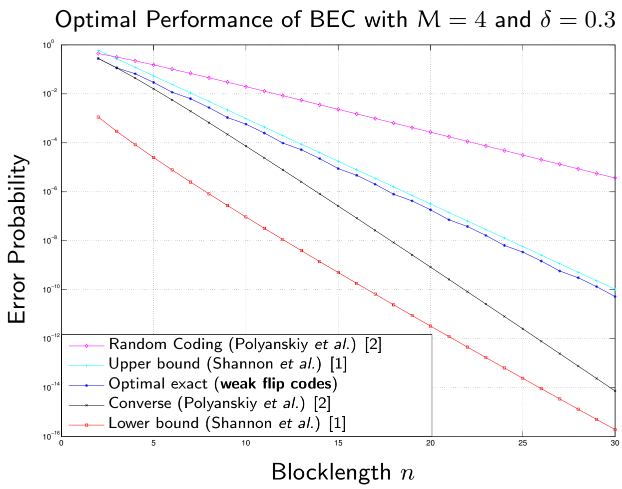


Figure 1: A comparison of some known bounds and the exact error probability of the globally optimal code with $\mathrm{M} = 4$ codewords on a BEC.

References

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