Optimal Finite Blocklength Code Design on the Binary Erasure Channel

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1 Introduction

In traditional coding theory, it is the goal to find good codes that operate close to the ultimate limit of the channel capacity as introduced by Shannon. In this work we would like to break away from these traditional simplifications and instead focus on an optimal (i.e., minimum error probability) design of codes for a certain given channel and a given fixed blocklength. Since for very short blocklength, it is not realistic to transmit large quantities of information, we start by looking at codes with only a few codewords, so called ultra-small block-codes. We introduce a new class of codes, called fair weak flip codes (the number of zeros and ones are almost equal under certain conditions on the blocklength n) and prove that they have beautiful quasi-linear properties.

2 Channel Models

We consider the binary erasure channel (BEC) with erasure probability δ:

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0 0 1 1
1 1 0 0
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3 Optimal Code Design

Define the following fair weak flip codes:

\[ C_{\text{BEC}}^{(5,10)} \triangleq \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0
\end{pmatrix} \]

\[ C_{\text{BEC}}^{(6,10)} \triangleq \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0
\end{pmatrix} \]

**Theorem 1.** For a BEC and for any n being a multiple of 10, an optimal codebook with \( M = 5 \) or \( M = 6 \) codewords is

\[ (C_{\text{BEC}}^{(M,10)} \cdot C_{\text{BEC}}^{(M,10)} \cdot \ldots \cdot C_{\text{BEC}}^{(M,10)})^{\times (n \mod 10=0)} \]

4 Optimal Exact Error Probability

![Graph showing optimal performance of BEC with M = 4 and δ = 0.3](image)

Figure 1: A comparison of some known bounds and the exact error probability of the globally optimal code with \( M = 4 \) codewords on a BEC.

References
