On the Fading Number of Multiple-Input Single-Output Fading Channels with Memory

Stefan M. Moser*
Department of Communication Engineering
National Chiao Tung University (NCTU)
Hsinchu, Taiwan
Email: stefan.moser@ieee.org

KEYWORDS: Beam-forming, channel capacity, fading number, flat fading, high SNR, memory, MISO, multiple-antenna, non-coherent.

1 The Channel Model

We consider a multiple-input single-output (MISO) fading channel:

\[ Y_k = H_k^* x_k + Z_k \]

where

- \( Y_k \in \mathbb{C} \) denotes the time-\( k \) channel output random variable;
- \( x_k \in \mathbb{C}^{n_T} \) denotes the time-\( k \) channel input vector satisfying either a peak-power or an average-power constraint;
- \( \{Z_k\} \sim \text{IID } \mathcal{N}(0, \sigma^2) \) denotes IID zero-mean white Gaussian noise of variance \( \sigma^2 > 0 \);
- \( H_k \) denotes the time-\( k \) fading vector of general (not necessarily Gaussian!) law including memory: we only assume that it is stationary, ergodic, of finite second moment \( E[\|H_k\|^2] < \infty \), and of finite differential entropy rate \( h(\{H_k\}) > -\infty \) (the regularity assumption). The realization of \( H \) is unknown both at transmitter and receiver (non-coherent situation).

2 Channel Capacity

We know that for such regular, non-coherent fading channels the capacity grows double-logarithmically at very high power (SNR \( \rightarrow \infty \)):

\[
C(\text{SNR}) = \log(1 + \log(1 + \text{SNR})) + \chi(\{H_k\}) + o(1)
\]

where \( o(1) \) tends to zero as \( \text{SNR} \rightarrow \infty \) and where \( \chi \) is a constant called fading number that is independent of the SNR. See Figure 1 for the example of Rician fading.

3 Main Result

**Theorem 1.** The MISO fading number with memory \( \chi(\{H_k^\ell\}) \) is upper-bounded by

\[
\chi(\{H_k^\ell\}) \leq \sup_{\hat{x}_{\ell} \sim U} \left\{ \log \pi + E[|H_0^\ell \hat{x}_0|^2] - h(H_0^\ell \hat{x}_0 | \{H_k^\ell \hat{x}_k\}_{k=\ell}^{-1}) \right\}
\]

and lower-bounded by

\[
\chi(\{H_k^\ell\}) \geq \sup_{\hat{x}} \left\{ \log \pi + E[|H_0^\ell \hat{x}_0|^2] - h(H_0^\ell \hat{x}_0 | \{H_k^\ell \hat{x}_k\}_{k=\ell}^{-1}) \right\}
\]

where \( \hat{x}_\ell \triangleq \frac{x_\ell}{|x_\ell|} \) denote vectors of unit length. Moreover, the lower bound is achievable by beam-forming: product of a constant unit vector \( \hat{x} \in \mathbb{C}^{n_T} \) (the beam-direction) and a circularly symmetric, scalar, complex IID random process \( \{X_k\} \) such that \( \log |X_k|^2 \sim \mathcal{U}([\log \log E, \log E]) \).

Figure 1: Bounds on the channel capacity of a SISO Rician fading channel with line-of-sight component \( d = 8 \).