On the Fading Number of Multiple-Input Single-Output Fading Channels with Memory

Stefan M. Moser*

Department of Communication Engineering

National Chiao Tung University (NCTU)

Hsinchu, Taiwan

Email: stefan.moser@ieee.org

KEYWORDS: Beam-forming, channel capacity, fading number, flat fading, high SNR, memory, MISO, multiple-antenna, non-coherent.

1 The Channel Model

We consider a multiple-input single-output (MISO) fading channel:

$$Y_k = \mathbf{H}_k^{\mathsf{T}} \mathbf{x}_k + Z_k$$

where

- $Y_k \in \mathbb{C}$ denotes the time-k channel output random variable;
- $\mathbf{x}_k \in \mathbb{C}^{n_{\mathrm{T}}}$ denotes the time-k channel input vector satisfying either a peak-power or an average-power constraint;
- $\{Z_k\} \sim$ IID $\mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ denotes IID zero-mean white Gaussian noise of variance $\sigma^2 > 0$;
- $\mathbf{H}_k$ denotes the time-k fading vector of general (not necessarily Gaussian!) law including memory: we only assume that it is stationary, ergodic, of finite second moment $\mathsf{E}\left[\|\mathbf{H}_k\|^2\right] < \infty$, and of finite differential entropy rate $h(\{\mathbf{H}_k\}) > -\infty$ (the *regularity* assumption). The realization of $\mathbf{H}$ is *unknown* both at transmitter and receiver (*non-coherent situation*).

2 Channel Capacity

We know that for such regular, non-coherent fading channels the capacity grows double-logarithmically at very high power ($\textsc{snr} \to \infty$):

$$C(\textsc{snr}) = \log(1 + \log(1 + \textsc{snr})) + \chi(\{\mathbf{H}_k\}) + o(1)$$

where $o(1)$ tends to zero as $\textsc{snr} \to \infty$ and where χ is a constant called **fading number** that is independent of the SNR. See Figure 1 for the example of Rician fading.

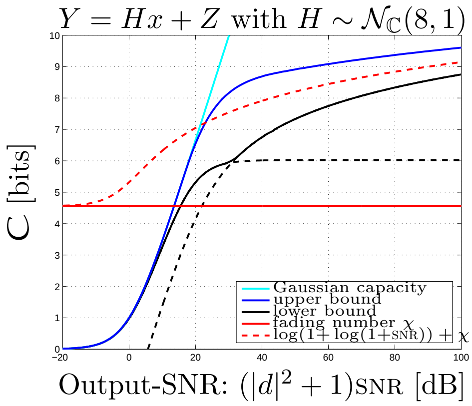

Figure 1: Bounds on the channel capacity of a SISO Rician fading channel with line-of-sight component $d = 8$.

3 Main Result

Theorem 1. *The MISO fading number with memory* $\chi(\{\mathbf{H}_k^{\mathsf{T}}\})$ *is upper-bounded by*

$$\chi(\{\mathbf{H}_k^{\mathsf{T}}\}) \le \sup_{\hat{\mathbf{x}}_{-\infty}^{0}} \Big\{ \log \pi + \mathsf{E}\left[\log |\mathbf{H}_0^{\mathsf{T}} \hat{\mathbf{x}}_0|^2\right] - h\left(\mathbf{H}_0^{\mathsf{T}} \hat{\mathbf{x}}_0 \,\middle|\, \{\mathbf{H}_\ell^{\mathsf{T}} \hat{\mathbf{x}}_\ell\}_{\ell=-\infty}^{-1}\right) \Big\}$$

and lower-bounded by

$$\chi(\{\mathbf{H}_k^{\mathsf{T}}\}) \ge \sup_{\hat{\mathbf{x}}} \Big\{ \log \pi + \mathsf{E}\left[\log |\mathbf{H}_0^{\mathsf{T}} \hat{\mathbf{x}}|^2\right] - h\left(\mathbf{H}_0^{\mathsf{T}} \hat{\mathbf{x}} \,\middle|\, \{\mathbf{H}_\ell^{\mathsf{T}} \hat{\mathbf{x}}\}_{\ell=-\infty}^{-1}\right) \Big\}$$

where $\hat{\mathbf{x}}_\ell \triangleq \frac{\mathbf{x}_\ell}{\|\mathbf{x}_\ell\|}$ *denote vectors of unit length. Moreover, the lower bound is achievable by* ***beam-forming****: product of a constant unit vector* $\hat{\mathbf{x}} \in \mathbb{C}^{n_{\mathrm{T}}}$ *(the beam-direction) and a circularly symmetric, scalar, complex IID random process* $\{X_k\}$ *such that* $\log |X_k|^2 \sim \mathcal{U}\left([\log \log \mathcal{E}, \log \mathcal{E}]\right)$.

---

*This work was supported in part by the National Science Council under NSC 94-2218-E-009-037.