The Fading Number of Multiple-Input–Multiple-Output Fading Channels with Memory

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1 The Channel Model

We consider a multiple-input–multiple-output (MIMO) fading channel with memory:

\[ Y_k = H_k x_k + Z_k \]

where

- \( Y_k \in \mathbb{C}^{n_R} \) denotes the time-\( k \) channel output random vector (\( n_R \) receive antennas);
- \( \{ Z_k \} \) IID \( \sim \mathcal{N}_\mathbb{C}(0, \sigma^2) \) denotes additive white (complex) Gaussian noise;
- \( x_k \in \mathbb{C}^{n_T} \) denotes the time-\( k \) channel input vector (\( n_T \) transmit antennas) satisfying either a peak-power or an average-power constraint \( \epsilon \) with signal-to-noise ratio \( \text{SNR} = \frac{\sigma^2}{\epsilon} \);
- \( H_k \) denotes the time-\( k \) random \( n_R \times n_T \) fading matrix of general (not necessarily Gaussian!) law including memory: we only assume that \( \{ H_k \} \) is stationary, ergodic, of finite second moment \( \mathbb{E}[\| H_k \|^2] < \infty \), and of finite differential entropy rate \( h(\{ H_k \}) > -\infty \) (the regularity assumption). The realizations of \( \{ H_k \} \) are unknown both at transmitter and receiver (noncoherent decoding).

2 Channel Capacity

We know that for such regular, noncoherent fading channels the capacity grows double-logarithmically at very high power (\( \text{SNR} \to \infty \)):

\[ C(\text{SNR}) = \log(1 + \log(1 + \text{SNR})) + \chi(\{ H_k \}) + o(1) \]

where \( o(1) \) tends to zero as \( \text{SNR} \to \infty \) and where \( \chi(\{ H_k \}) \) is a constant called fading number that is independent of the SNR.

3 Main Results

**Theorem 1.** Assume some general stationary channel model with input \( x_k \in \mathbb{C}^{n_T} \) and output \( Y_k \in \mathbb{C}^{n_R} \). Then under very mild conditions on the channel law and apart from edge effects the capacity-achieving input distribution can be assumed to be stationary.

**Lemma 2.** The capacity-achieving input process of the channel model in Section 1 can be assumed to be circularly symmetric, i.e., the input \( \{ X_k \} \) can be replaced by \( \{ X_k e^{i \theta_k} \} \), where \( \{ \theta_k \} \) is IID \( \sim \mathcal{U}([0, 2\pi]) \) and independent of every other random quantity.

**Theorem 3.** The MIMO fading number with memory \( \chi(\{ H_k \}) \) is given by

\[
\chi(\{ H_k \}) = \sup_{Q(\{ X_k \}) \text{ stationary, circ. sym.}} \left\{ h_\lambda \left( \frac{\mathbb{E}_0 X_0 \mathbb{E}_0 X_0}{\| \mathbb{E}_0 X_0 \|^2} \right) \left( \frac{\mathbb{E}_0 X_0 \mathbb{E}_0 X_\ell}{\| \mathbb{E}_0 X_\ell \|^2} \right) \right\}_{\ell = -\infty}^{\ell = \infty} + n_R \mathbb{E} \left( \log \| \mathbb{E}_0 X_0 \|^2 \right) - 2 - h(\mathbb{E}_0 X_0) \left( \mathbb{E}_\ell X_\ell \right)_{\ell = -\infty}^{\ell = \infty}
\]

where the maximization is over all stochastic unit-vector processes \( \{ X_k \} \) that are stationary and circularly symmetric.

References