Bounds on the Capacity of the Additive Inverse Gaussian Noise Channel

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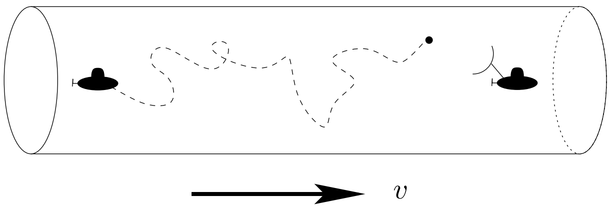
1 Nano Devices in Fluid Medium



- nano devices communicate by **exchange of molecular particles**
- fluid medium flows with **constant drift velocity** v
- particles suffer from **Brownian motion**: modeled as Wiener process with parameter σ^2
- information is encoded in **release time**

$\Longrightarrow$ inverse problem to Wiener process: **inverse Gaussian distribution**

2 Channel Model

- transmitter at fixed position $w = 0$
- receiver at fixed position $w = d$ (normalized to $d = 1$)
- **travel time** N is random with an **inverse Gaussian distribution**: $N \sim \mathrm{IG}(\mu, \lambda)$ with average travel time $\mu \triangleq \frac{d}{v}$, and Brownian motion parameter $\lambda \triangleq \frac{d^2}{\sigma^2}$. PDF:

$$f_N(n) = \sqrt{\frac{\lambda}{2\pi n^3}} \exp\left(-\frac{\lambda(n-\mu)^2}{2\mu^2 n}\right) \mathrm{I}\{n > 0\}$$

- channel input: release time x
- channel output: arrival time $Y = x + N$
- implicit nonnegativity constraint: $X \geq 0$
- average-delay constraint: $\mathsf{E}[X] \leq m$

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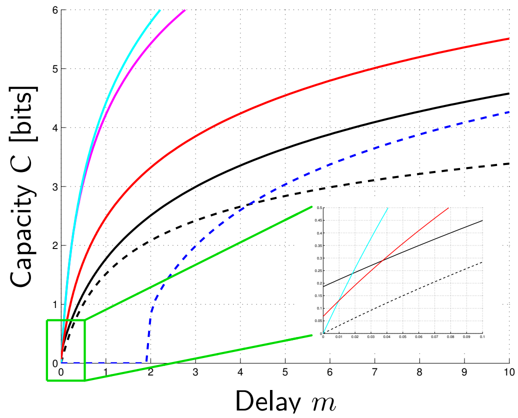
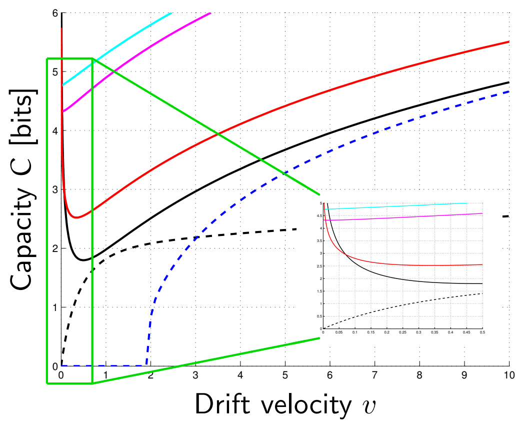
3 New Bounds on Capacity





4 Exact Asymptotic Capacity

$$\mathrm{C}(m) = \log m + \frac{1}{2}\log\frac{\lambda e}{2\pi\mu^3} - \frac{3}{2}e^{\frac{2\lambda}{\mu}}\,\mathrm{Ei}\left(-\frac{2\lambda}{\mu}\right) + o(1)$$

$$\mathrm{C}(v) = \frac{3}{2}\log v + \frac{1}{2}\log\frac{\lambda m^2 e}{2\pi} + o(1)$$

5 References

[1] K. V. Srinivas, R. S. Adve, and A. W. Eckford, "Molecular communication in fluid media: The additive inverse Gaussian noise channel," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4678–4692, July 2012.