On Ultra-Short Block-Codes on the Binary Asymmetric Channel

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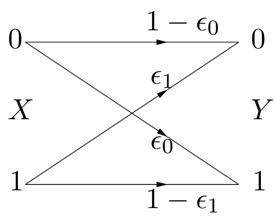
1 Introduction

Shannon proved in his ground-breaking work that it is possible to find an information transmission scheme that can transmit messages at arbitrarily small error probability as long as the transmission rate in **bits per channel use** is below the so-called **capacity** of the channel and the blocklength is very large. However, a large blocklength might not be an option and might have to be restricted to some reasonable size. **The question now arises what we can theoretically say about the performance of communication systems with such restricted blocksize and about good design for such codes.**

To simplify the problem, we currently focus on binary channels and start with the simplest type of communication: **a code with only two possible, equally likely messages**.

2 Channel Model

We consider a general binary channel **binary asymmetric channel (BAC)** with crossover probabilities that are not identical:



Without loss of generality we can restrict the values of these parameters as follows:

$$0 \le \epsilon_0 \le \epsilon_1 \le 1, \quad \epsilon_0 \le 1-\epsilon_0, \quad \epsilon_0 \le 1-\epsilon_1.$$

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3 Main Results

Definition 1. *We define the* ***flip-flop code of type*** t *as follows: for every* $t \in \left\{0,1,\ldots,\left\lfloor\frac{n}{2}\right\rfloor\right\}$, *we have*

$$\mathcal{C}_t = \begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{pmatrix} \triangleq \begin{pmatrix}\mathbf{x}\\ \bar{\mathbf{x}}\end{pmatrix}$$

where

$$\mathbf{x}_1 = \mathbf{x} \triangleq 00\cdots 0\underbrace{11\cdots 1}_{w_\mathrm{H}(\mathbf{x})=t},$$
$$\mathbf{x}_2 = \bar{\mathbf{x}} \triangleq 11\cdots 100\cdots 0.$$

Proposition 2. *Fix the blocklength* n. *Then, irrespective of the BAC channel parameters* ϵ_0 *and* ϵ_1, ***there always exists a flip-flop code of type*** t, $\mathcal{C}_t$, ***for some choice of*** $0 \le t \le \left\lfloor\frac{n}{2}\right\rfloor$ ***that is optimal in the sense that it minimizes the error probability***.

In the situation of a flip-flop code of type t, $\mathcal{C}_t^{(n)}$, the **log likelihood ratio** $\mathrm{LLR}_t^{(n)}(\epsilon_0,\epsilon_1,d)$ is a function of the BAC channel parameters and of d, **the Hamming distance between the received vector and the *first* codeword**.

Proposition 3. *For a fixed flip-flop code* $\mathcal{C}_t^{(n)}$ *and a fixed BAC* (ϵ_0,ϵ_1), ***there exists a threshold*** ℓ, $t \le \ell \le \left\lfloor\frac{n-1}{2}\right\rfloor$, *such that the optimal ML decision rule can be stated as*

$$g(\mathbf{y}) = \begin{cases}\mathbf{x}_1 & \text{if } 0 \le d \le \ell,\\ \mathbf{x}_2 & \text{if } \ell+1 \le d \le n.\end{cases}$$

Theorem 4. *Fix blocklength* n. *Under a particular* ***fixed decision rule*** ℓ, *the* ***flip-flop codebook of type*** t ***is optimal*** *if* (ϵ_0,ϵ_1) *belongs to*

$$\left\{(\epsilon_0,\epsilon_1)\,\middle|\,\mathrm{LLR}_t^{(n-1)}(\epsilon_0,\epsilon_1,\ell) > 0 \wedge \mathrm{LLR}_{t-1}^{(n-1)}(\epsilon_0,\epsilon_1,\ell) < 0\right\}.$$

If the region is empty, then t *is not optimal for any BAC.*