Clarification of Equation (345) in Paper about MIMO Fading Number with Memory

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In [1, Appendix E] in the derivation of Equation (345) we rely on Lemma 12. This is not completely correct as our assumption here is only that $\hat{\mathbf{X}}_{-\kappa}^{0} \sim Q_{\mathsf{E},\epsilon}^{\kappa+1}$ where $Q_{\mathsf{E},\epsilon}^{\kappa+1}$ is quasi-stationary. We provide here a proof of (345) that does not rely on Lemma 12.

$$
\begin{aligned}
h\big(\{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{0} \,\big|\, \hat{\mathbf{X}}_{-\kappa}^{0}\big) &= \sum_{i=-\kappa}^{0} h\big(\mathbb{H}_i \hat{\mathbf{X}}_i \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{i-1}, \hat{\mathbf{X}}_{-\kappa}^{0}\big) && (1)\\
&= \sum_{i=-\kappa}^{0} h\big(\mathbb{H}_i \hat{\mathbf{X}}_i \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{i-1}, \hat{\mathbf{X}}_{-\kappa}^{i}\big) && (2)\\
&= \sum_{i=-\kappa}^{0} h\big(\mathbb{H}_0 \hat{\mathbf{X}}_0 \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa-i}^{-1}, \hat{\mathbf{X}}_{-\kappa-i}^{0}\big) && (3)\\
&\geq \sum_{i=-\kappa}^{0} h\big(\mathbb{H}_0 \hat{\mathbf{X}}_0 \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{-1}, \hat{\mathbf{X}}_{-\kappa}^{0}\big) && (4)\\
&= (\kappa+1) h\big(\mathbb{H}_0 \hat{\mathbf{X}}_0 \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{-1}, \hat{\mathbf{X}}_{-\kappa}^{0}\big). && (5)
\end{aligned}
$$

Here the first equality (1) follows from the chain rule; the subsequent equality (2) from the fact that conditional on $\hat{\mathbf{X}}_{-\kappa}^{i}$, $\hat{\mathbf{X}}_{i+1}^{0}$ is independent of all other random variables in the expression; in (3) we make use of our assumption that $\hat{\mathbf{X}}_{-\kappa}^{0}$ is quasi-stationary and that $\{\mathbb{H}_k\}$ is stationary; the inequality (4) follows from conditioning that cannot increase entropy: we add the random vectors $\{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{-\kappa-i-1}, \hat{\mathbf{X}}_{-\kappa}^{-\kappa-i-1}$ to the conditioning; and the final equality (5) follows from the fact that the terms in the sum do not depend on i.

Hence,

$$
h\big(\mathbb{H}_0 \hat{\mathbf{X}}_0 \,\big|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{-1}, \hat{\mathbf{X}}_{-\kappa}^{0}\big) \leq \frac{1}{\kappa+1} h\big(\{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\kappa}^{0} \,\big|\, \hat{\mathbf{X}}_{-\kappa}^{0}\big). \qquad (6)
$$

References

[1] S. M. Moser, "The fading number of multiple-input multiple-output fading channels with memory," *IEEE Transactions on Information Theory*, vol. 55, no. 6, pp. 2716–2755, June 2009.