The Fading Number of Multiple-Input–Multiple-Output Fading Channels with Memory

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1 The Channel Model

We consider a multiple-input–multiple-output (MIMO) fading channel with memory:

$$\mathbf{Y}_k = \mathbb{H}_k \mathbf{x}_k + \mathbf{Z}_k$$

where

- $\mathbf{Y}_k \in \mathbb{C}^{n_R}$ denotes the time-k channel output random vector (n_R **receive antennas**);
- $\{\mathbf{Z}_k\}$ IID $\sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{0}, \sigma^2 \mathsf{I}_{n_R}\right)$ denotes additive white (complex) **Gaussian noise**;
- $\mathbf{x}_k \in \mathbb{C}^{n_T}$ denotes the time-k channel input vector (n_T **transmit antennas**) satisfying either a peak-power or an average-power constraint $\mathcal{E}$ with signal-to-noise ratio $\textsc{snr} \triangleq \frac{\mathcal{E}}{\sigma^2}$;
- $\mathbb{H}_k$ denotes the time-k random $n_R \times n_T$ fading matrix of **general** (not necessarily Gaussian!) **law including memory**: we only assume that $\{\mathbb{H}_k\}$ **is stationary**, **ergodic**, of finite second moment $\mathsf{E}\left[\|\mathbb{H}_k\|_F^2\right] < \infty$, and of finite differential entropy rate $h(\{\mathbb{H}_k\}) > -\infty$ (the **regularity** assumption). The realizations of $\{\mathbb{H}_k\}$ are **unknown** both at transmitter and receiver (**noncoherent decoding**).

2 Channel Capacity

We know that for such regular, noncoherent fading channels the capacity grows double-logarithmically at very high power ($\textsc{snr} \to \infty$):

$$\mathsf{C}(\textsc{snr}) = \log(1 + \log(1 + \textsc{snr})) + \chi\left(\{\mathbb{H}_k\}\right) + o(1)$$

where $o(1)$ tends to zero as $\textsc{snr} \to \infty$ and where $\chi\left(\{\mathbb{H}_k\}\right)$ is a constant called **fading number** that is independent of the SNR.

3 Main Results

Theorem 1. *Assume some general stationary channel model with input* $\mathbf{x}_k \in \mathbb{C}^{n_T}$ *and output* $\mathbf{Y}_k \in \mathbb{C}^{n_R}$. *Then under very mild conditions on the channel law and apart from edge effects the* ***capacity-achieving input distribution*** *can be assumed to be* ***stationary***.

Lemma 2. *The capacity-achieving input process of the channel model in Section 1 can be assumed to be* ***circularly symmetric***, i.e., *the input* $\{\mathbf{X}_k\}$ *can be replaced by* $\{\mathbf{X}_k e^{i\Theta_k}\}$, *where* $\{\Theta_k\}$ *is IID* $\sim \mathcal{U}([0, 2\pi])$ *and independent of every other random quantity.*

Theorem 3. *The MIMO fading number with memory* $\chi\left(\{\mathbb{H}_k\}\right)$ *is given by*

$$\chi\left(\{\mathbb{H}_k\}\right) = \sup_{\substack{Q_{\{\hat{\mathbf{X}}_k\}} \\ \text{stationary} \\ \text{circ. sym.}}} \Bigg\{ h_\lambda\left(\frac{\mathbb{H}_0 \hat{\mathbf{X}}_0}{\|\mathbb{H}_0 \hat{\mathbf{X}}_0\|} \,\middle|\, \left\{ \frac{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell}{\|\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\|} \right\}_{\ell=-\infty}^{-1} \right) + n_R \mathsf{E}\left[\log \|\mathbb{H}_0 \hat{\mathbf{X}}_0\|^2\right] - \log 2 - h\left(\mathbb{H}_0 \hat{\mathbf{X}}_0 \,\middle|\, \{\mathbb{H}_\ell \hat{\mathbf{X}}_\ell\}_{\ell=-\infty}^{-1}, \hat{\mathbf{X}}_{-\infty}^0\right) \Bigg\}$$

where the maximization is over all ***stochastic unit-vector processes*** $\{\hat{\mathbf{X}}_k\}$ *that are* ***stationary*** *and* ***circularly symmetric***.

References

[1] S. M. Moser, "The fading number of multiple-input–multiple-output fading channels with memory," May 2008, to appear in *IEEE Transactions on Information Theory*. [Online]. Available: http://moser.cm.nctu.edu.tw/publications.html

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