



# Advanced Topics in Information Theory

Lecture Notes

Stefan M. Moser

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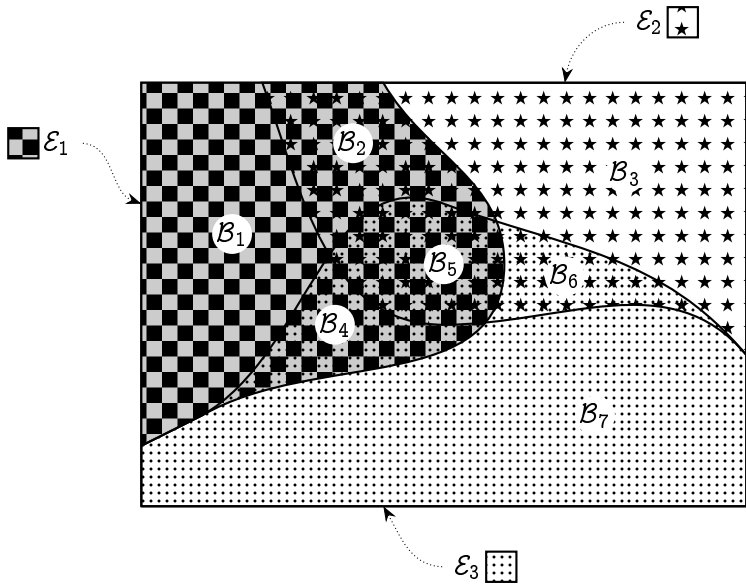
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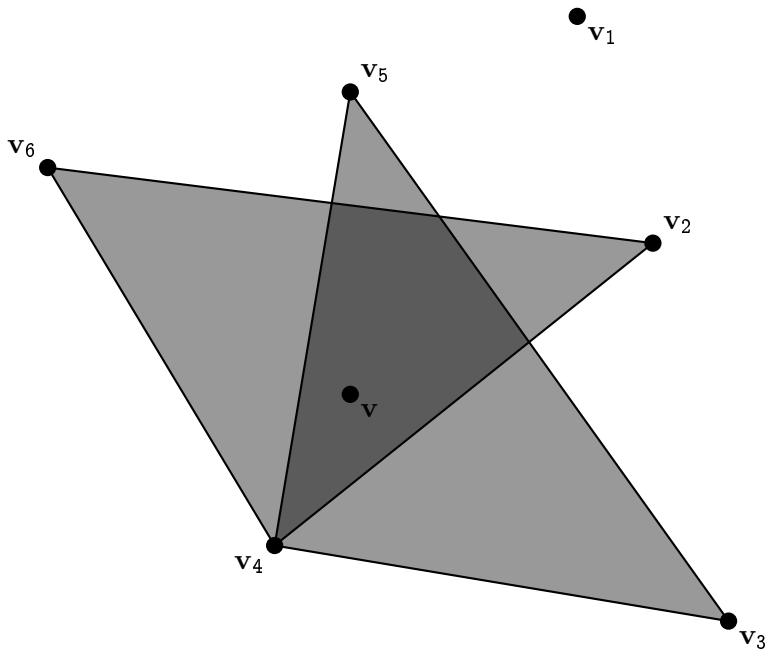
5<sup>th</sup> Edition — 2022.

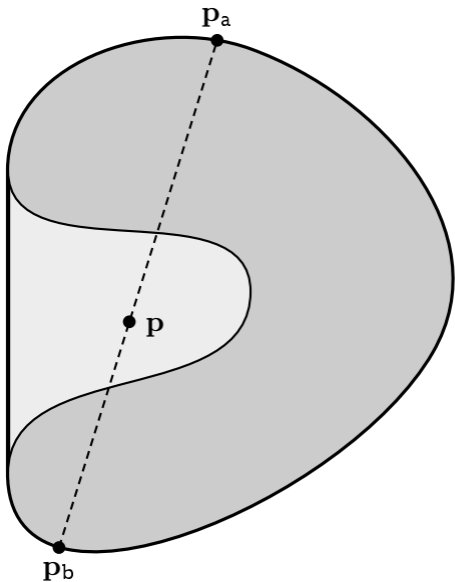
Version 5.7.

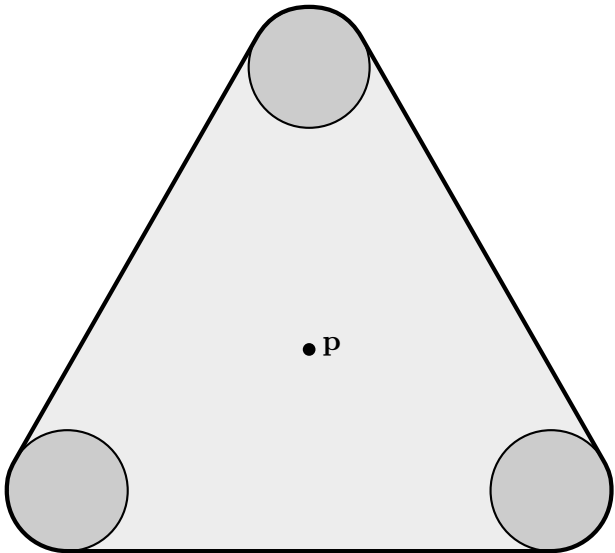
Compiled on 10 July 2024.

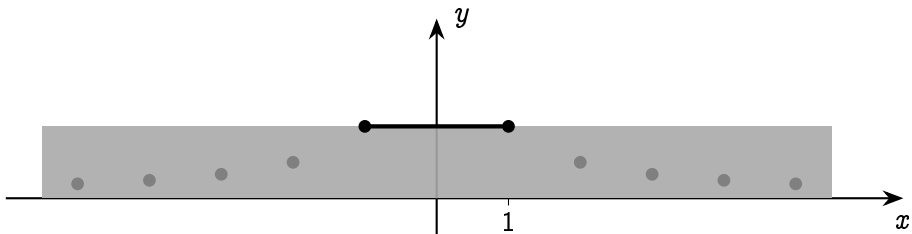
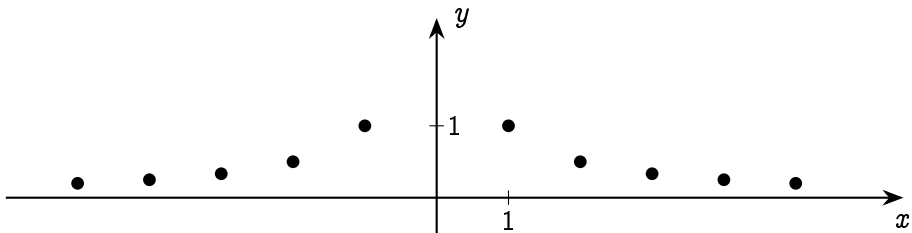
For the latest version see <https://moser-isi.ethz.ch/scripts.html>

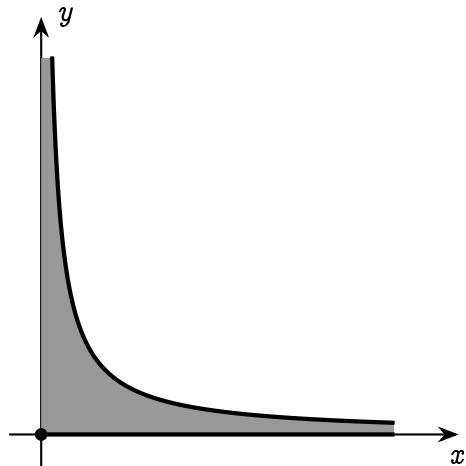
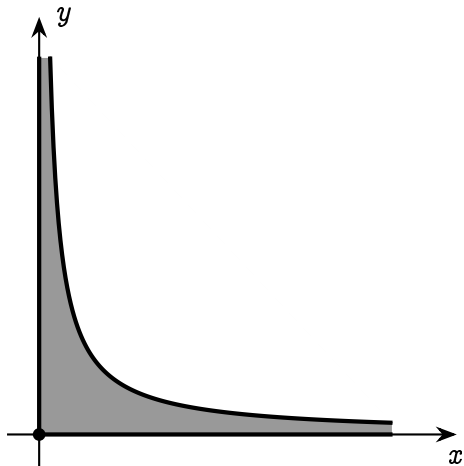


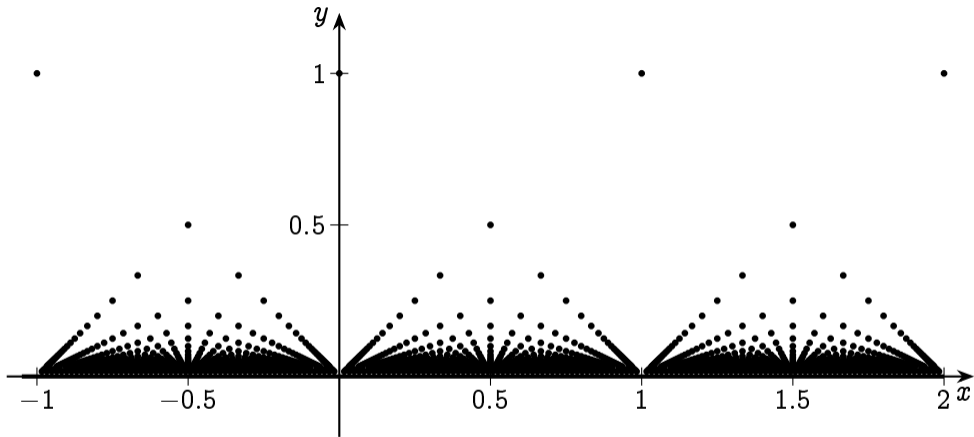




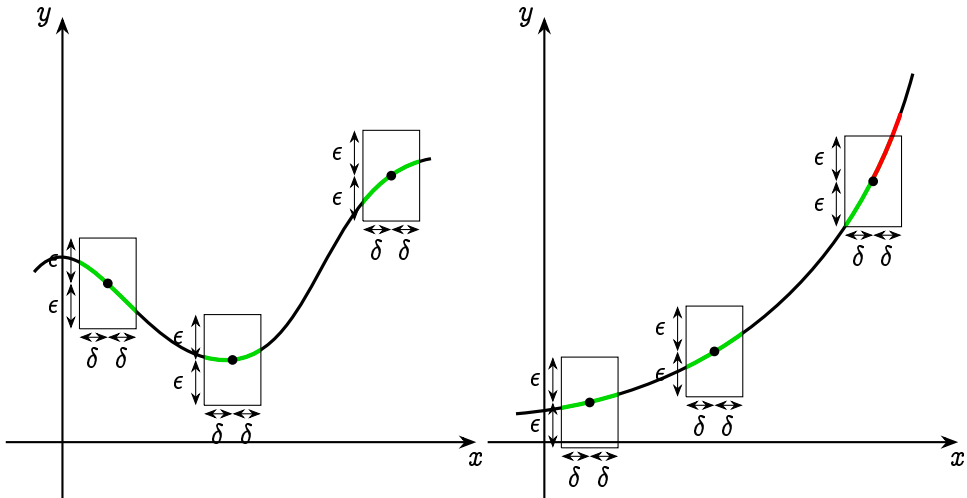


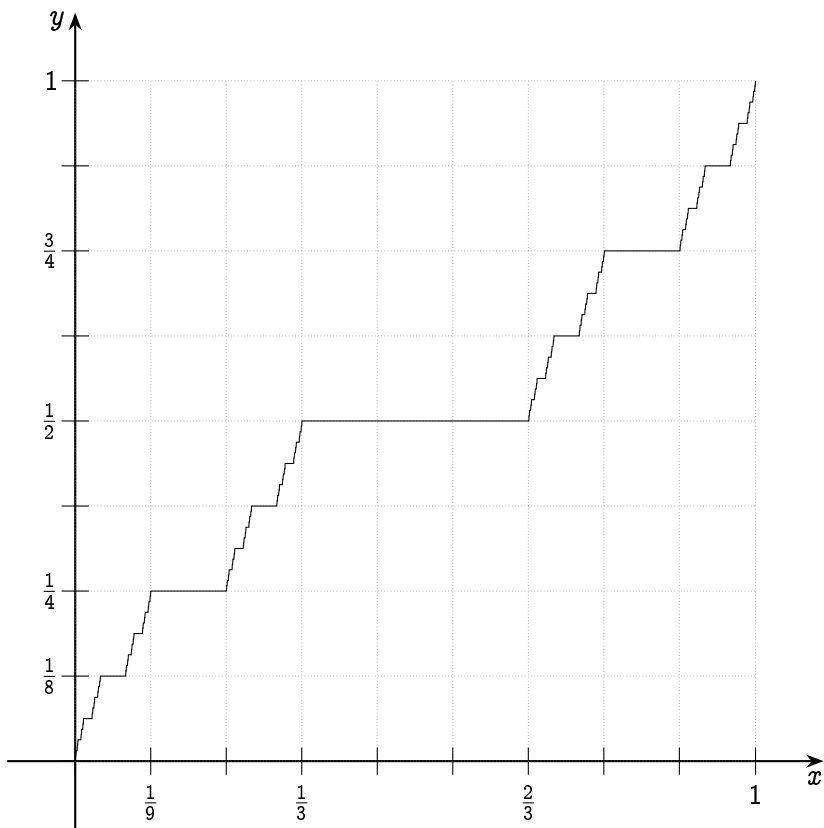


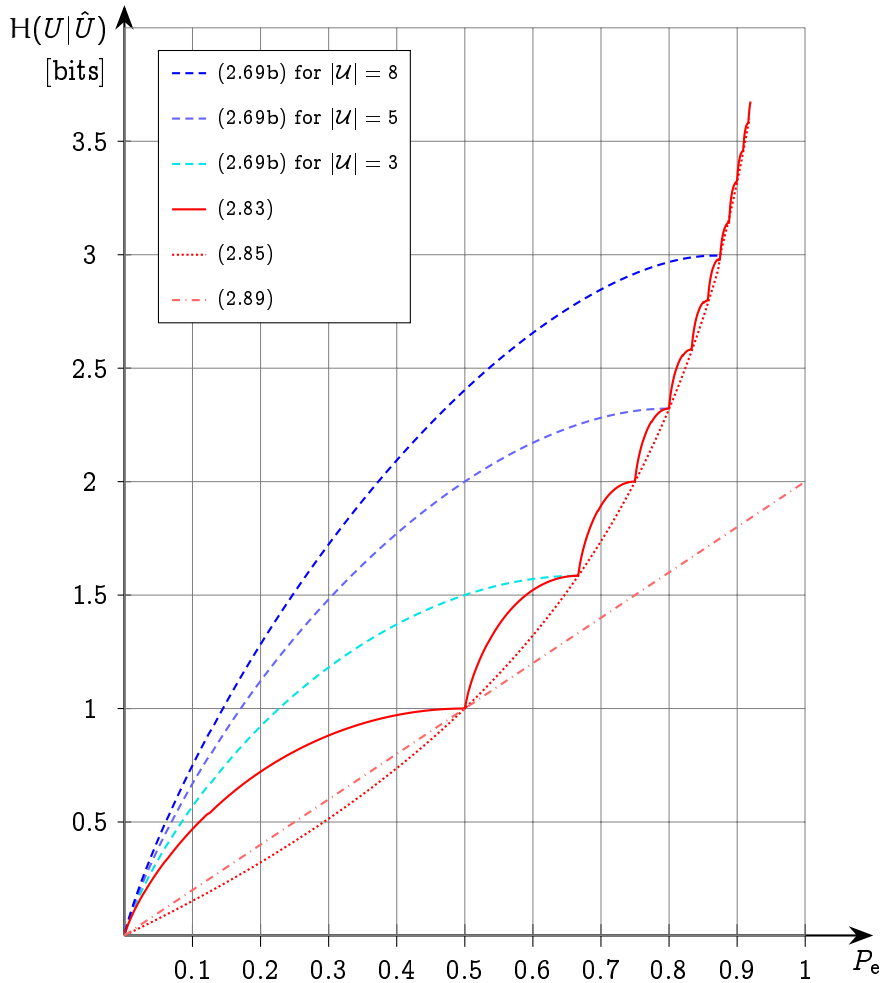


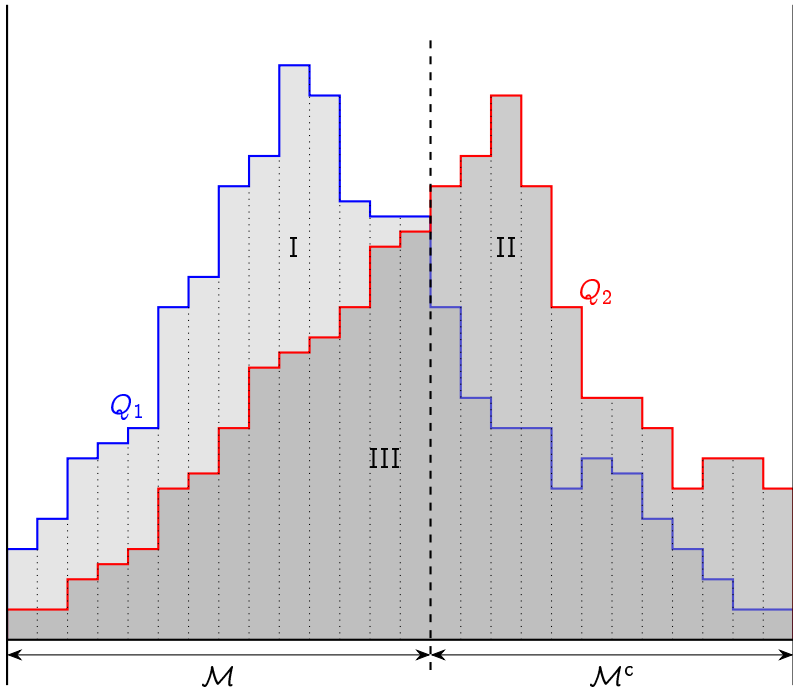


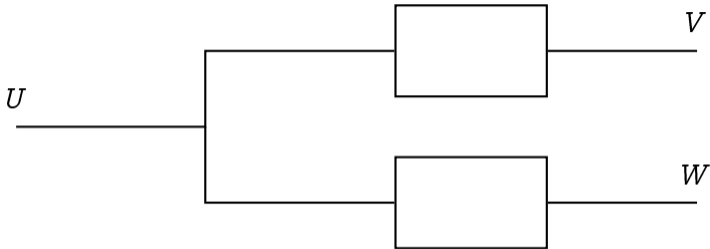


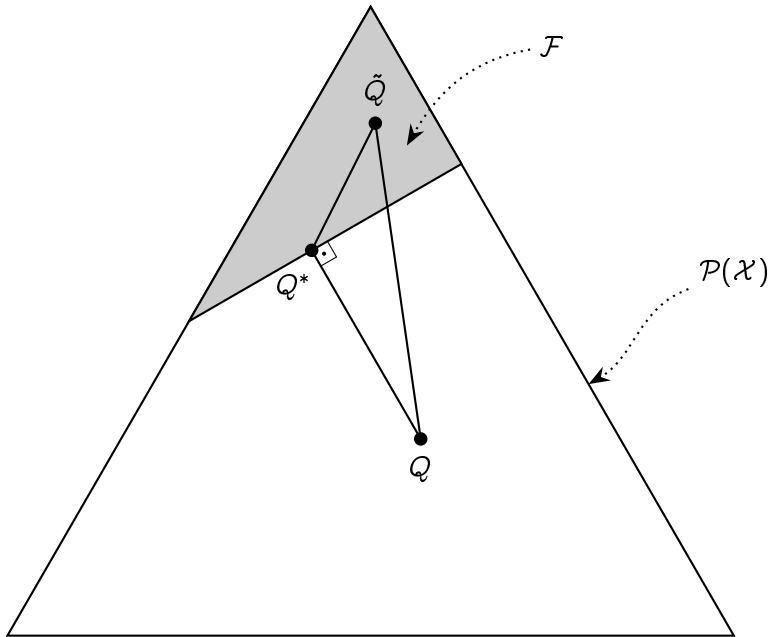


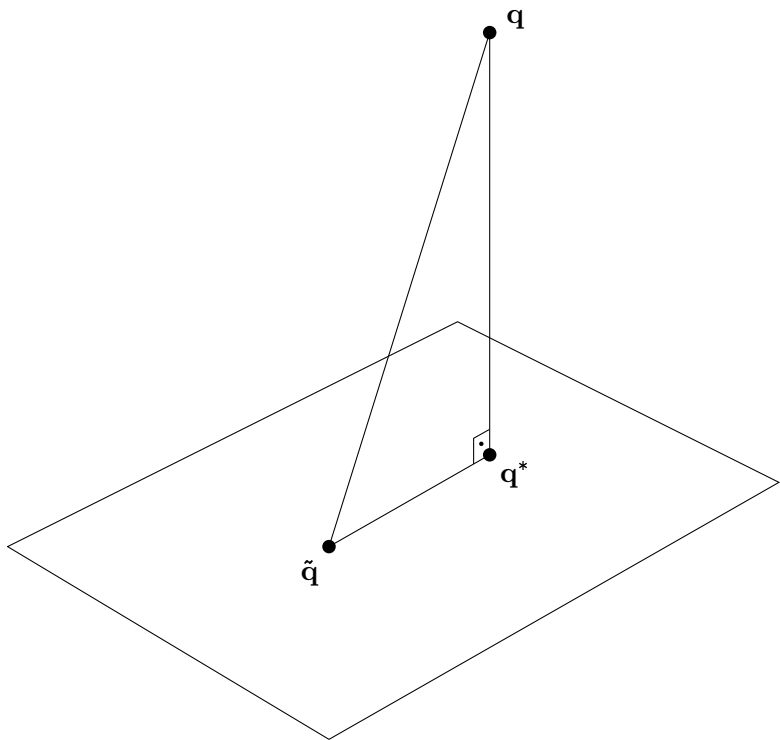


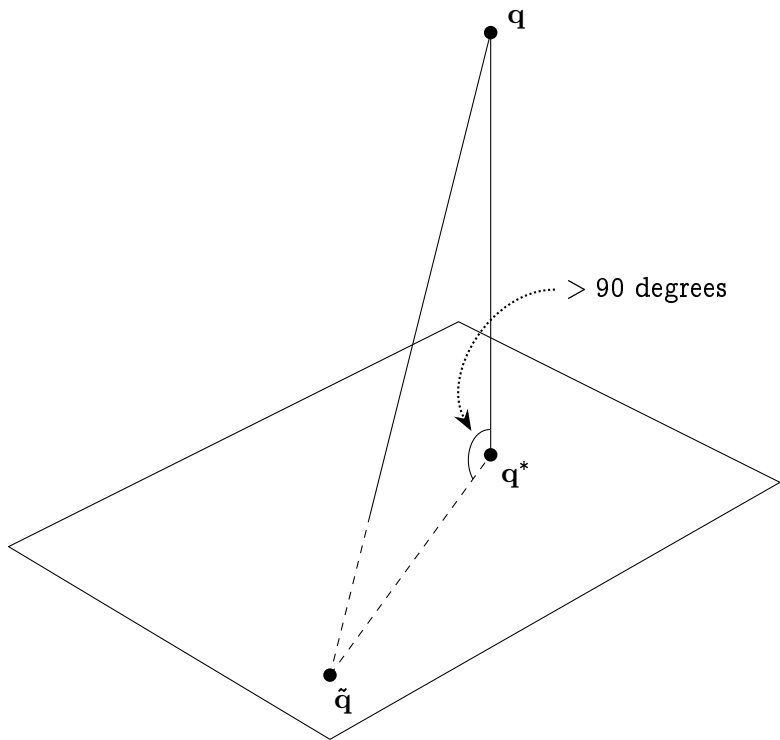




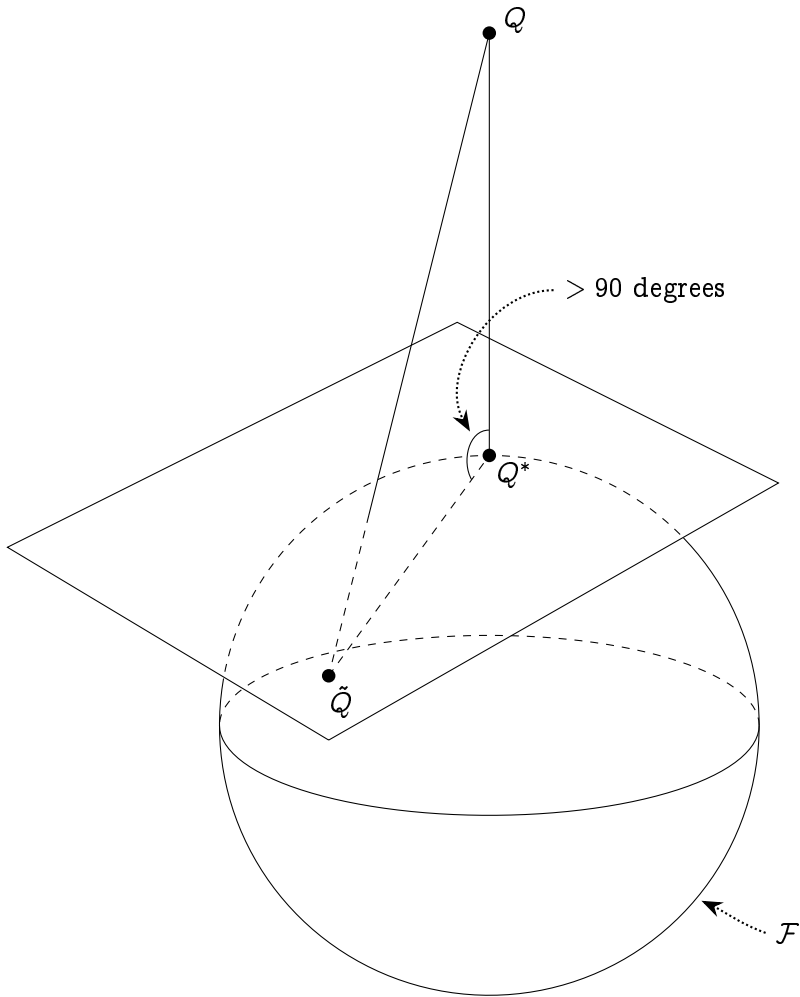


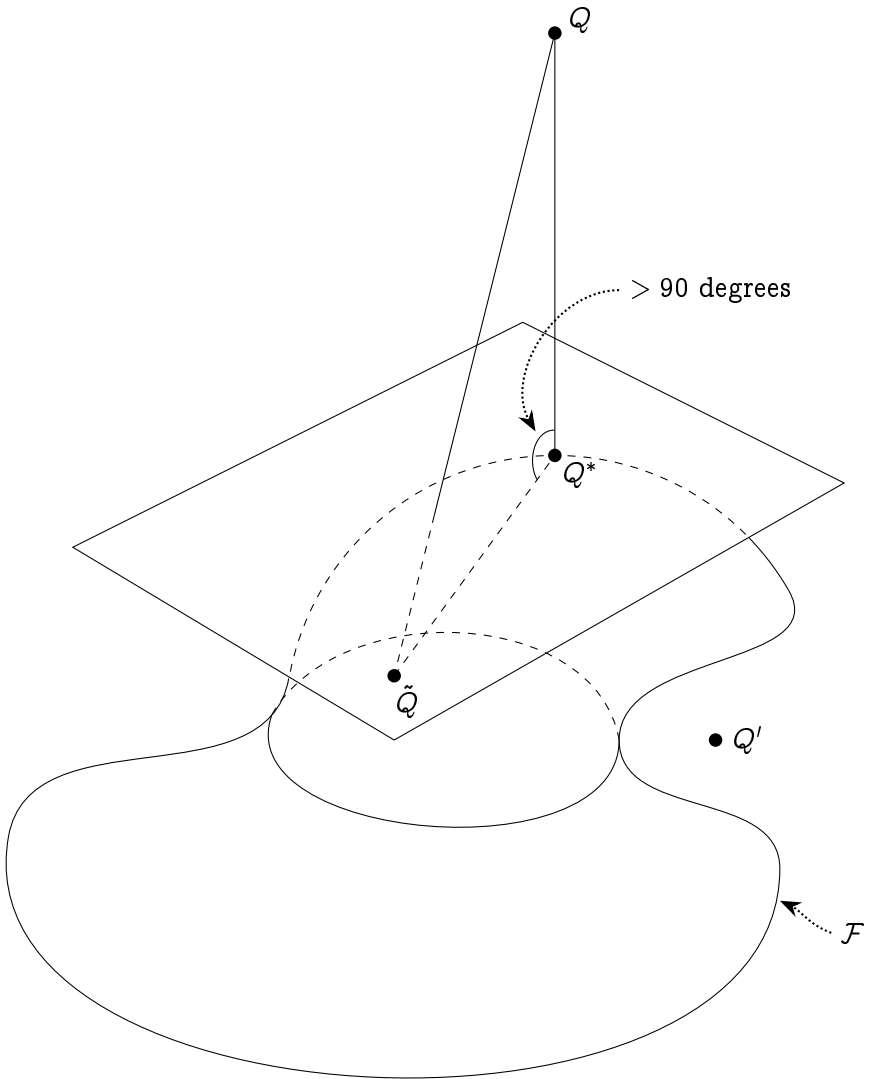


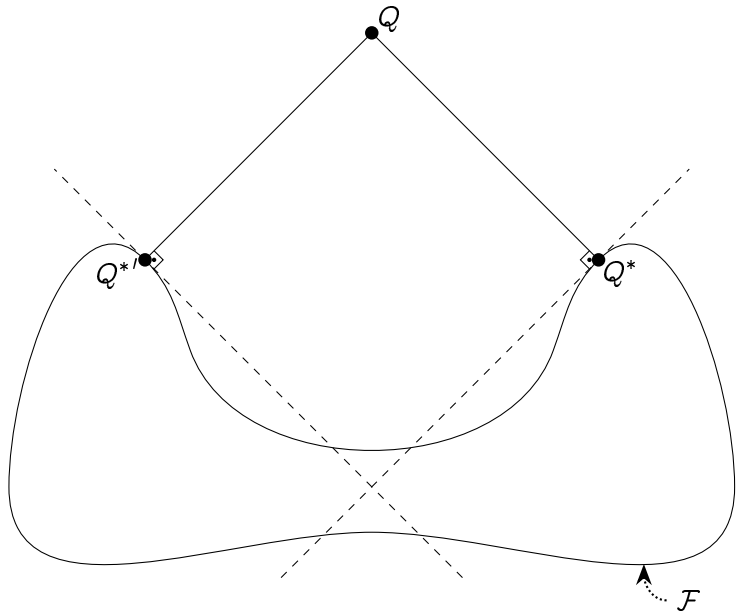


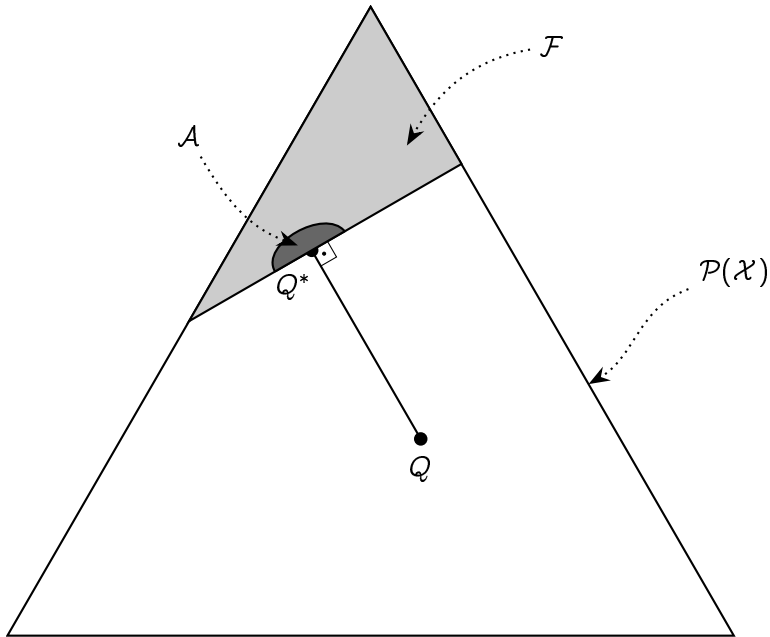


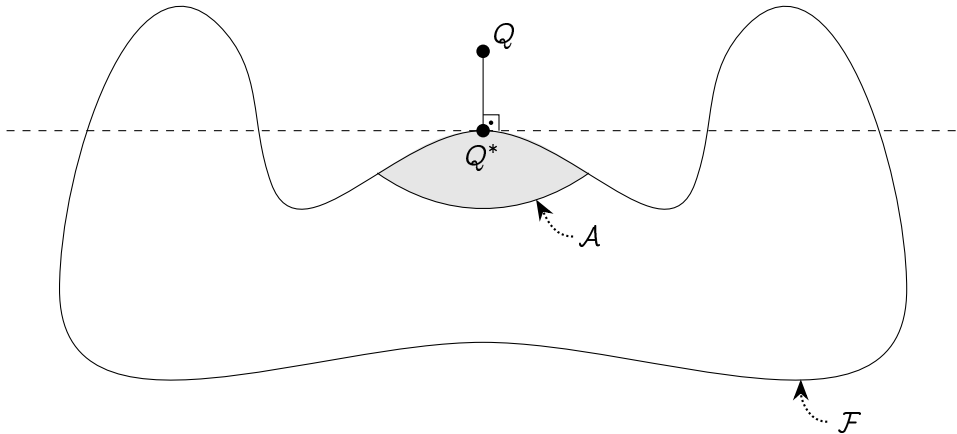


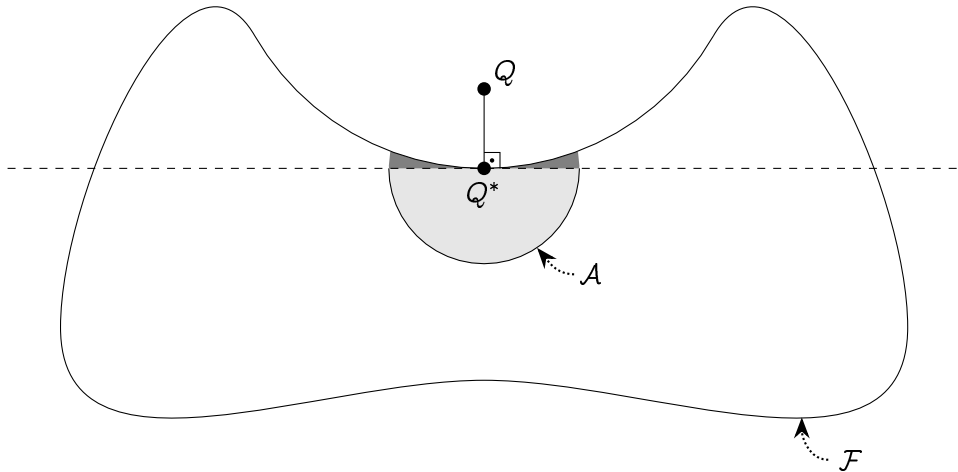


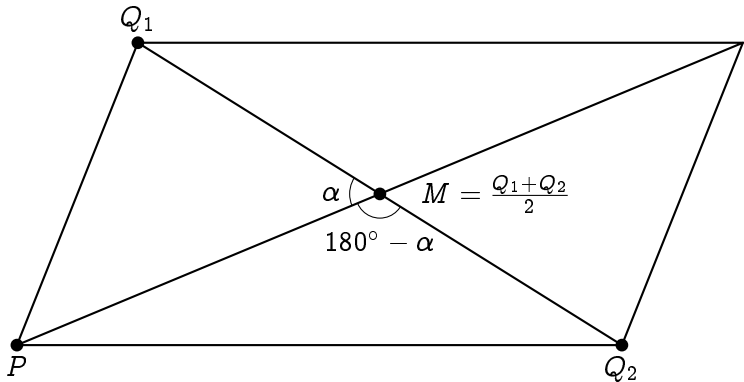


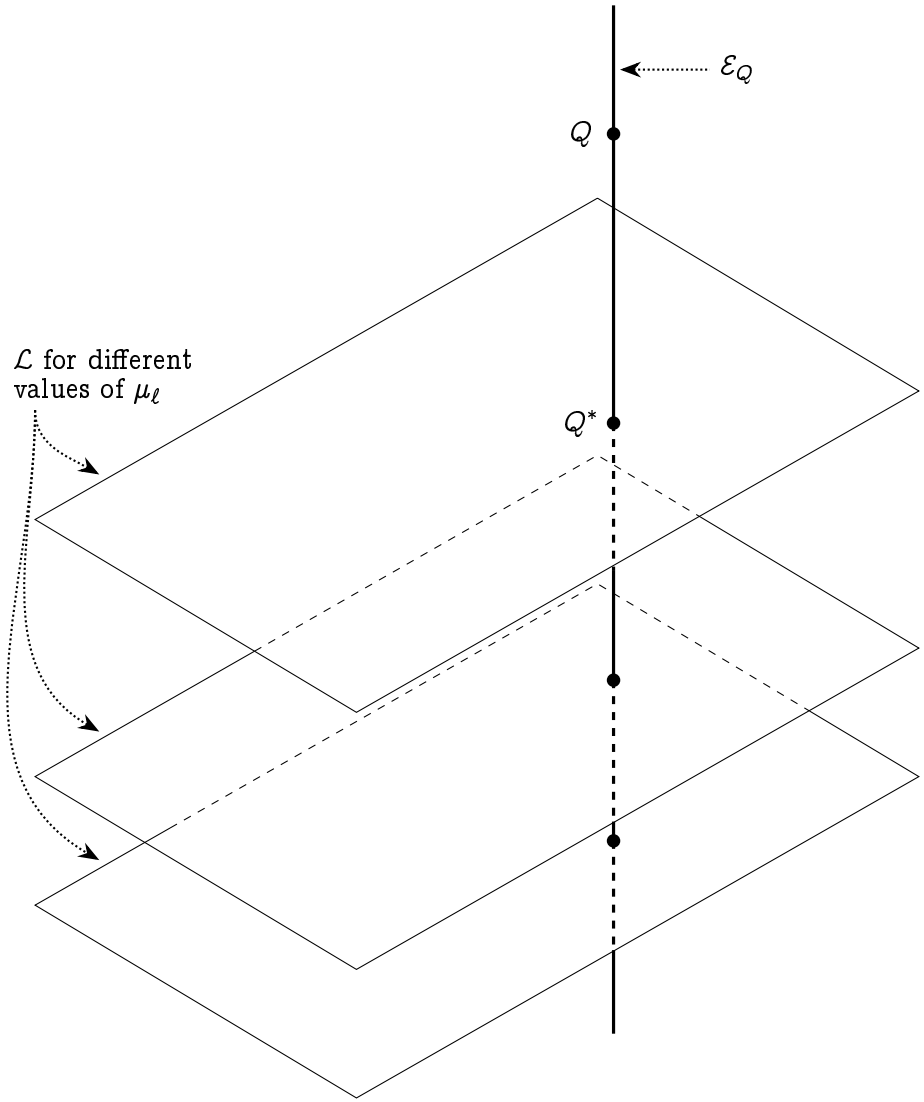




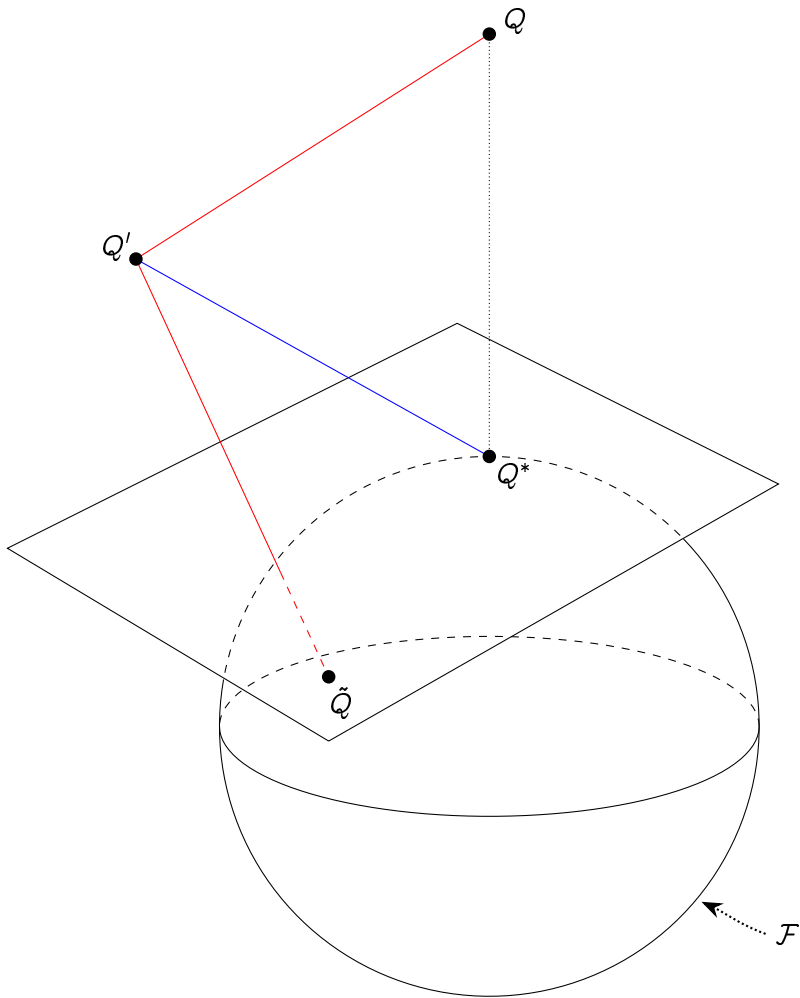


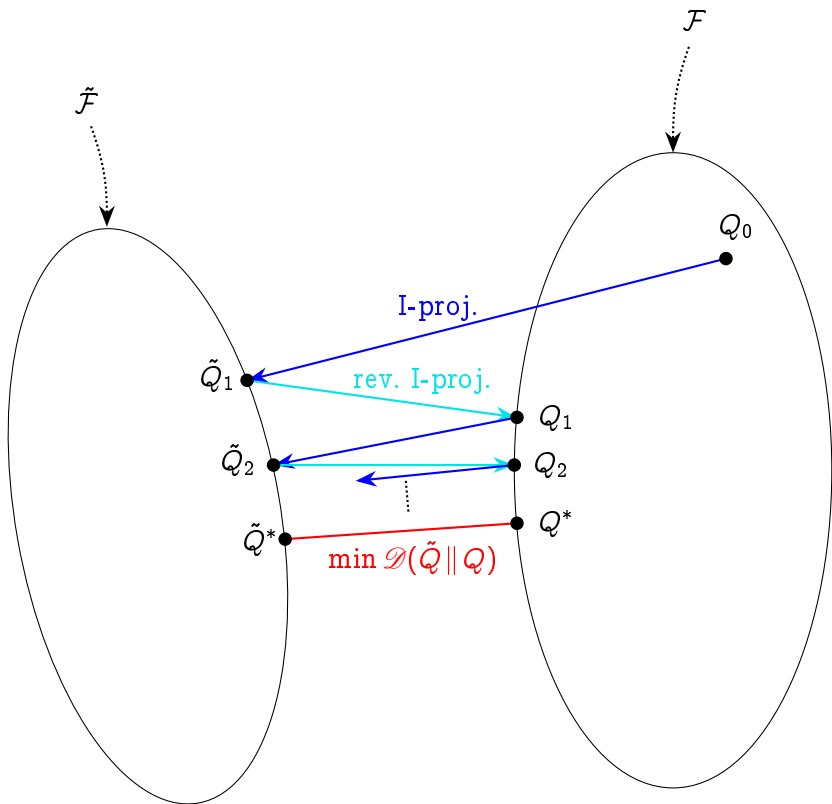


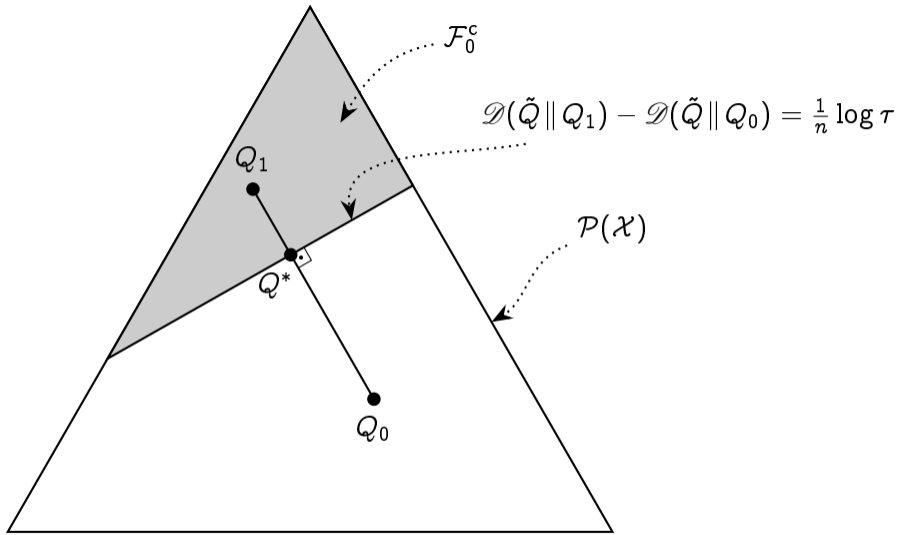


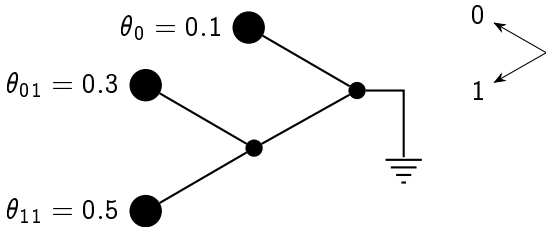


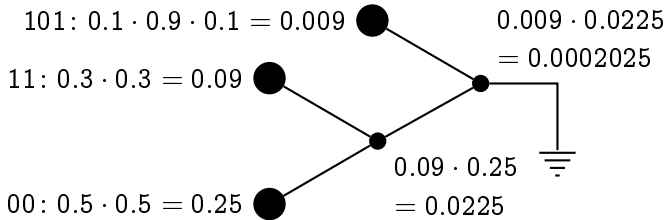


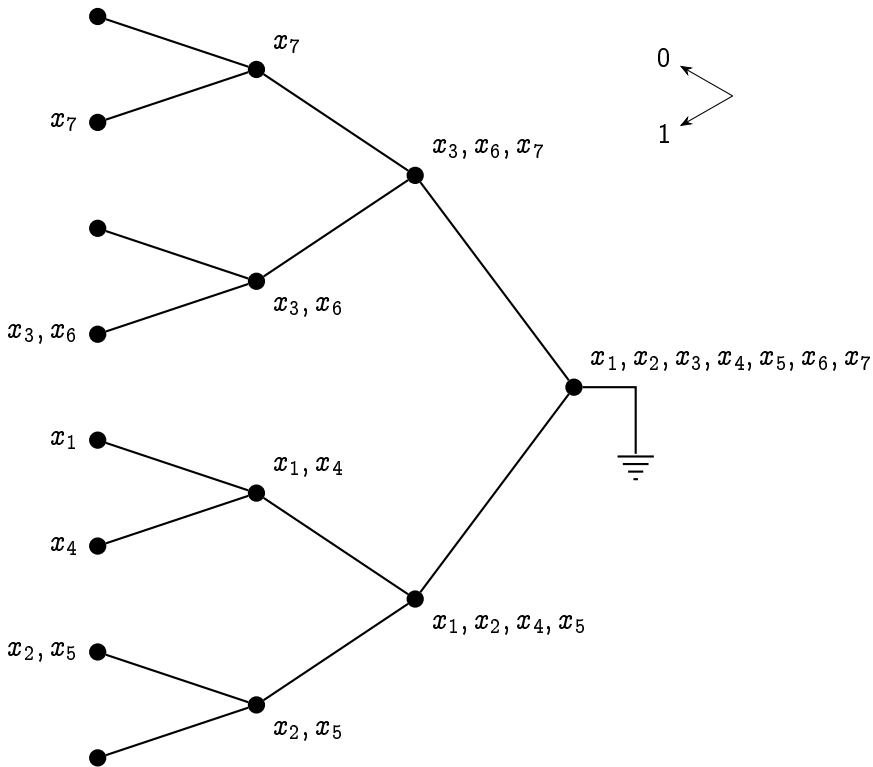


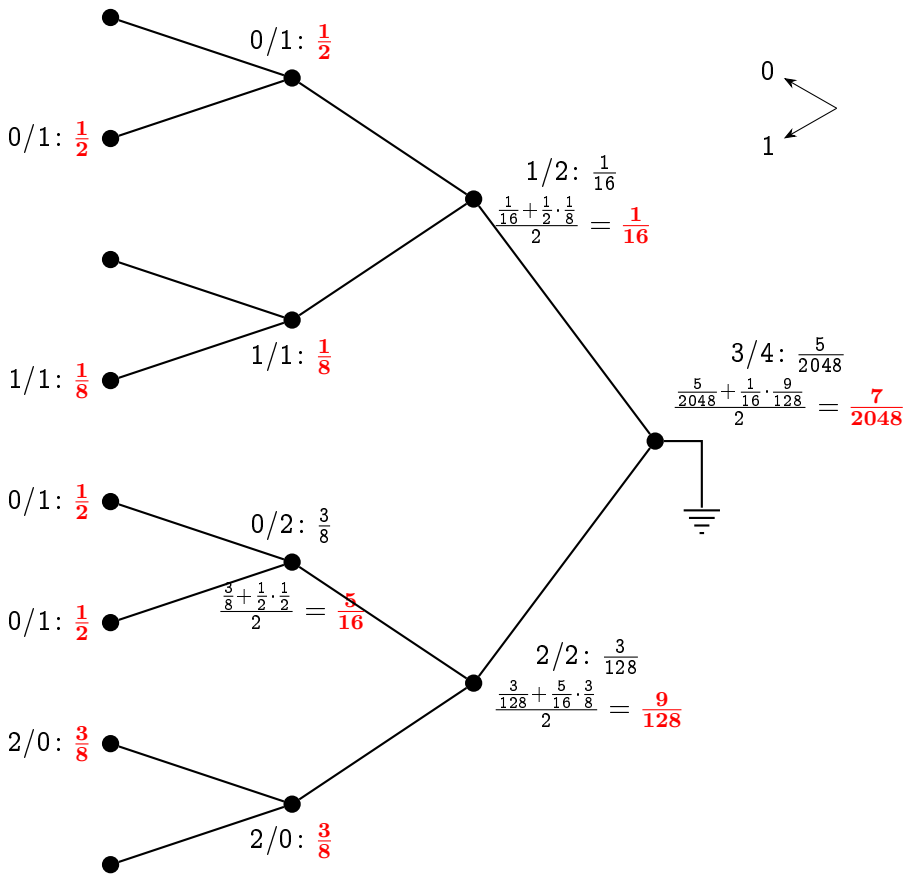


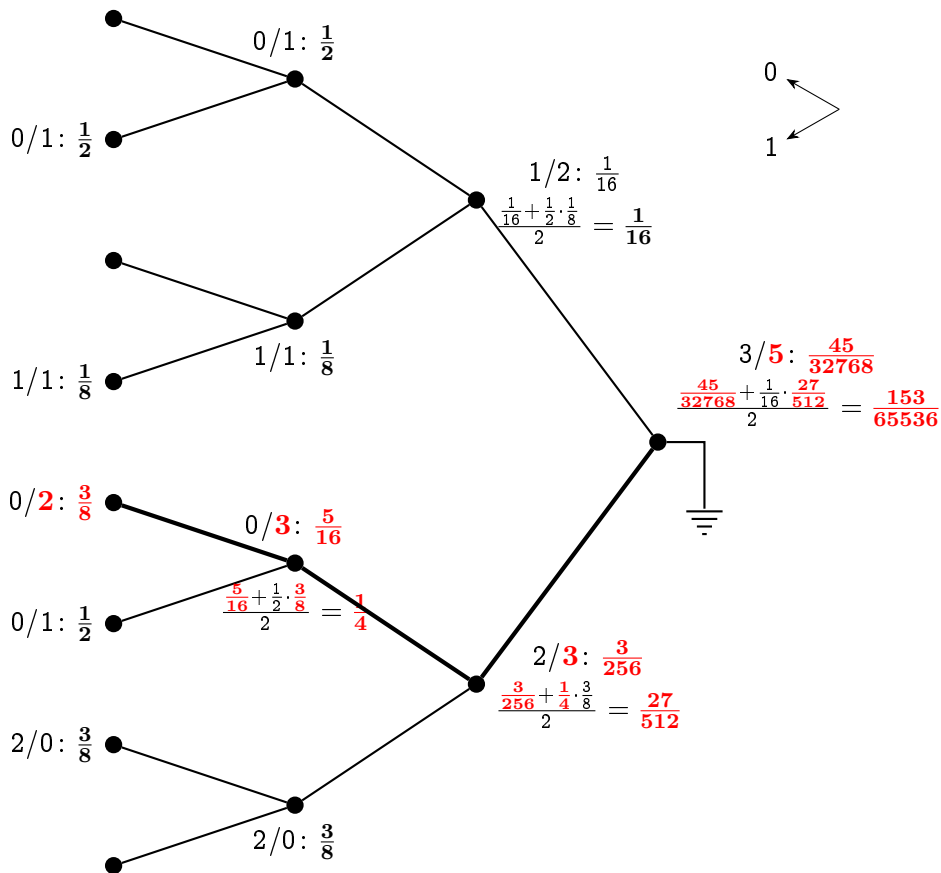




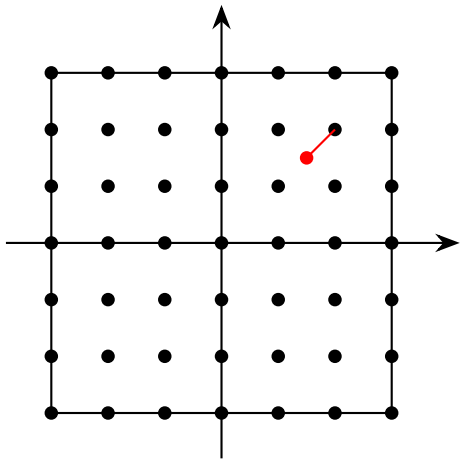




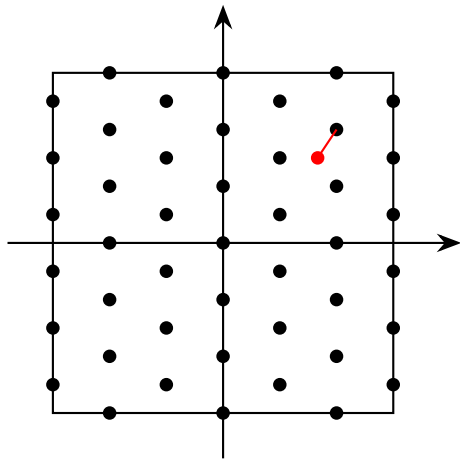




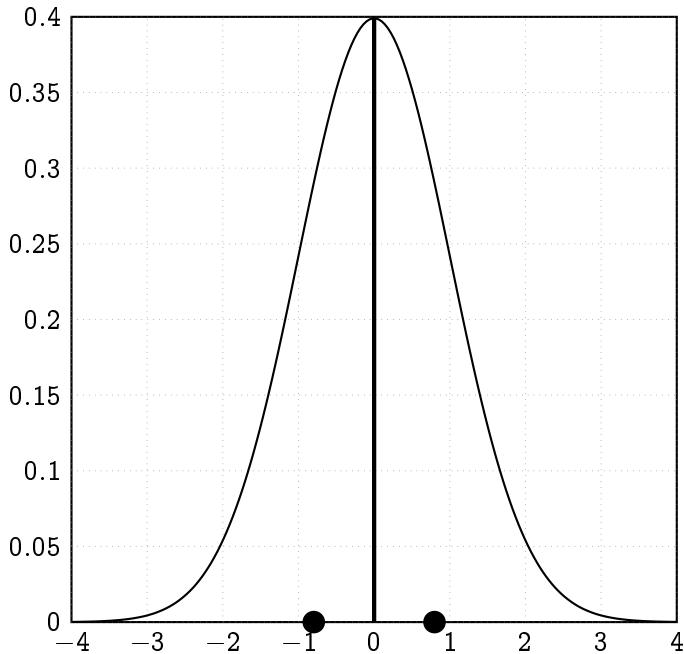




independent description  
49 points



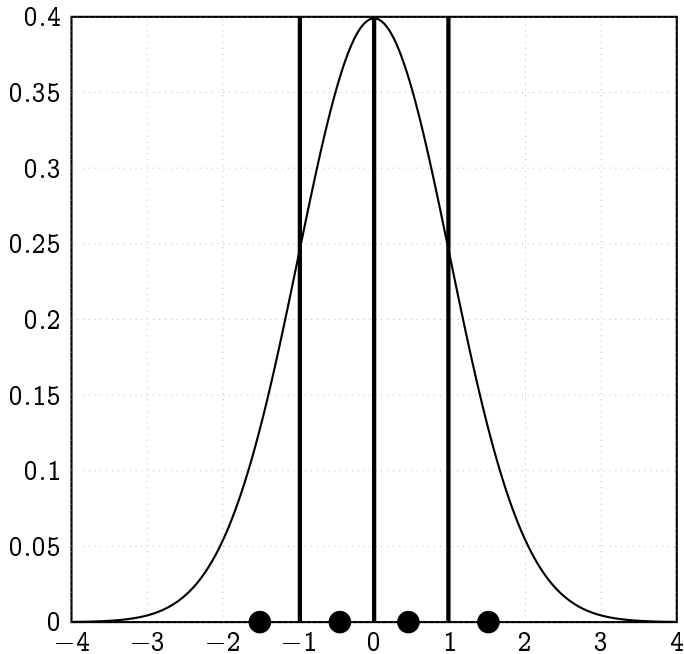
dependent description  
45 points

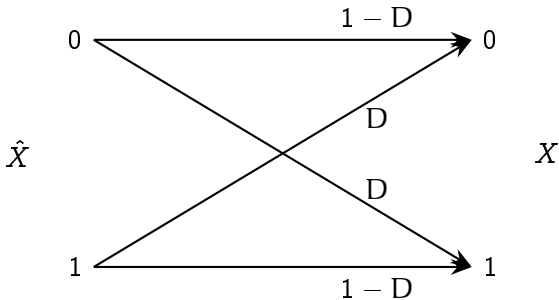


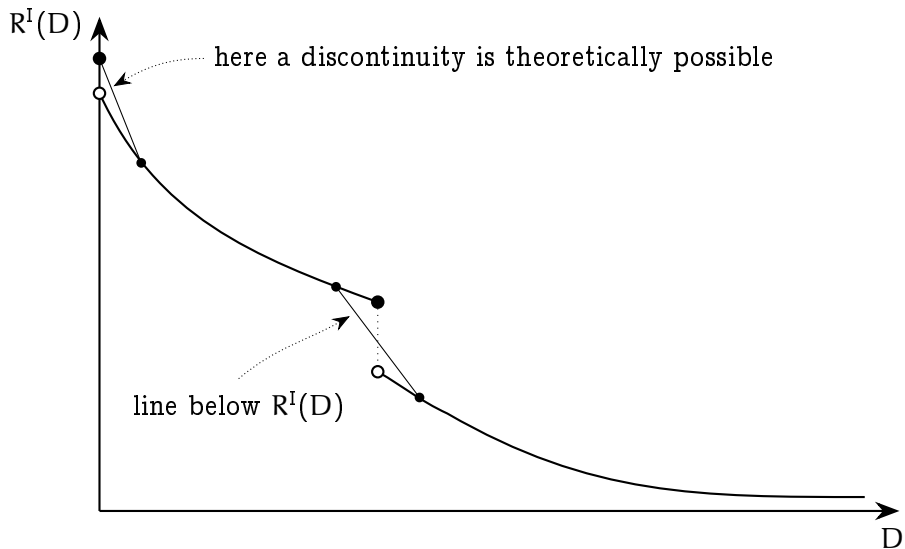
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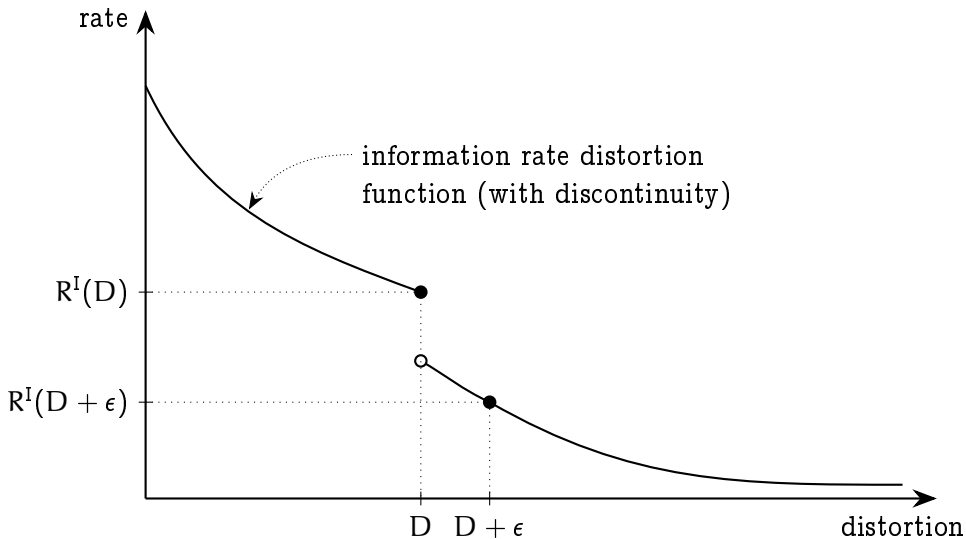
$\hat{x}_1$	$\hat{x}_2$	$\theta$
0.5	1	0.75
0.3578	1.3288	0.8433
0.3973	1.4011	0.8992
0.4202	1.4450	0.9326
0.4336	1.4714	0.9525
0.4414	1.4872	0.9643
0.4461	1.4966	0.9714
0.4488	1.5022	0.9755
⋮		
0.4528	1.5104	0.9816

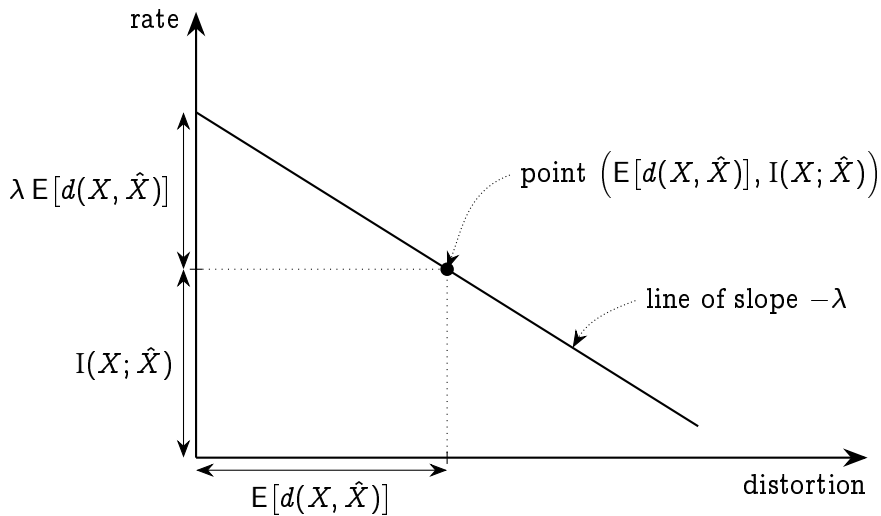
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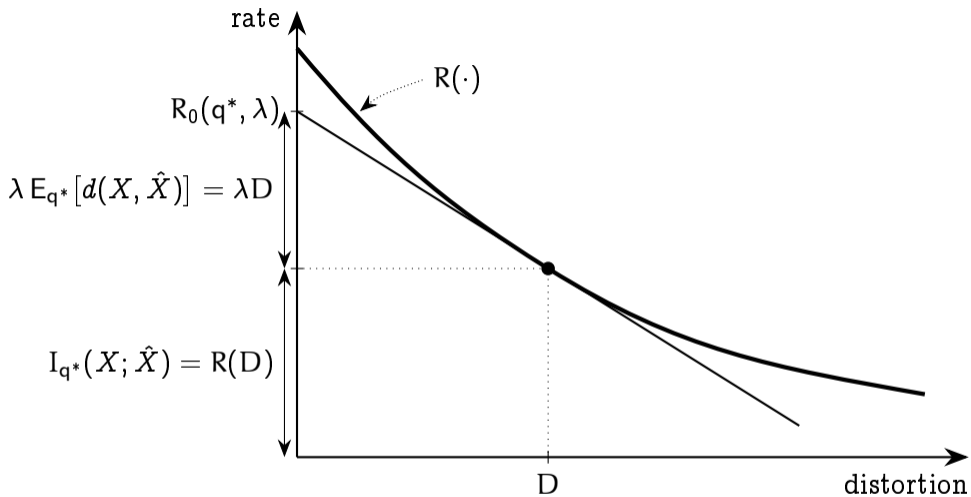


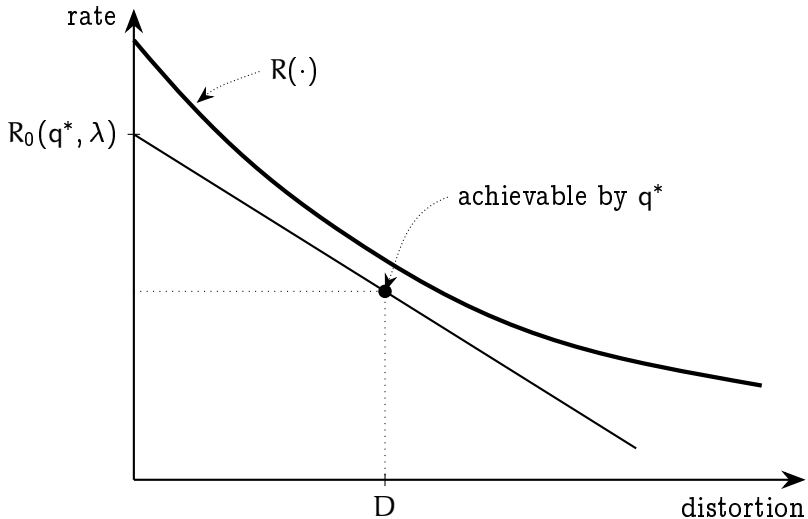


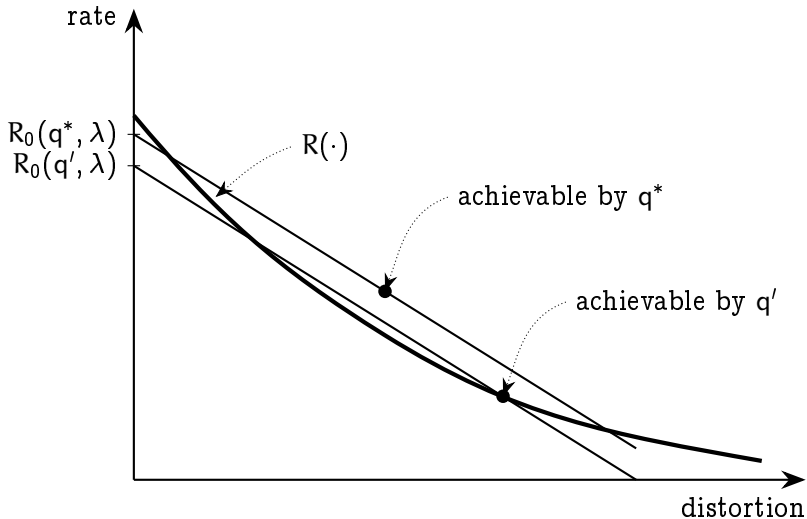


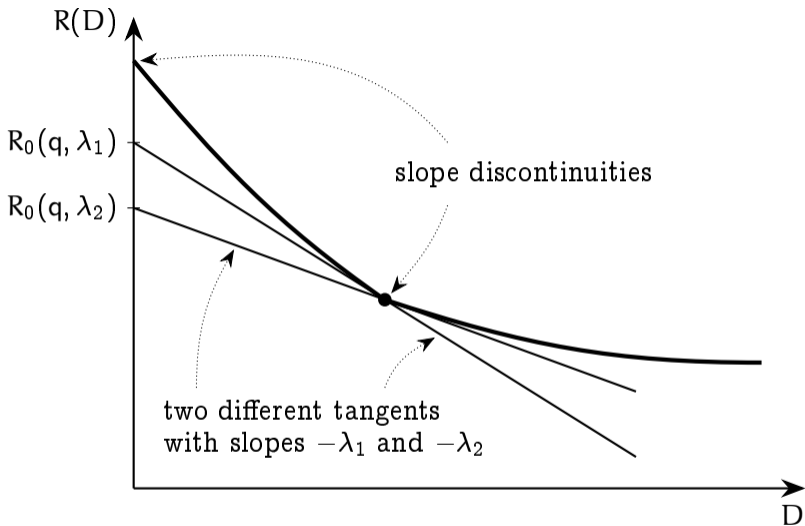


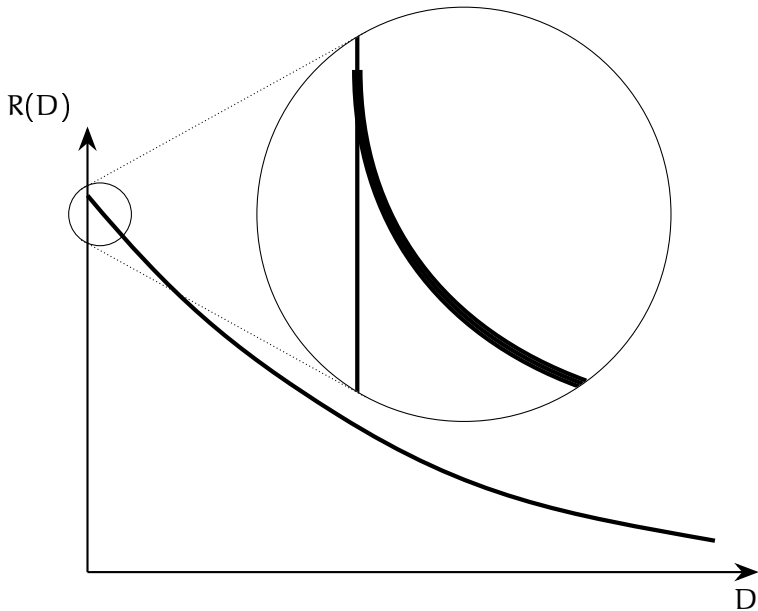






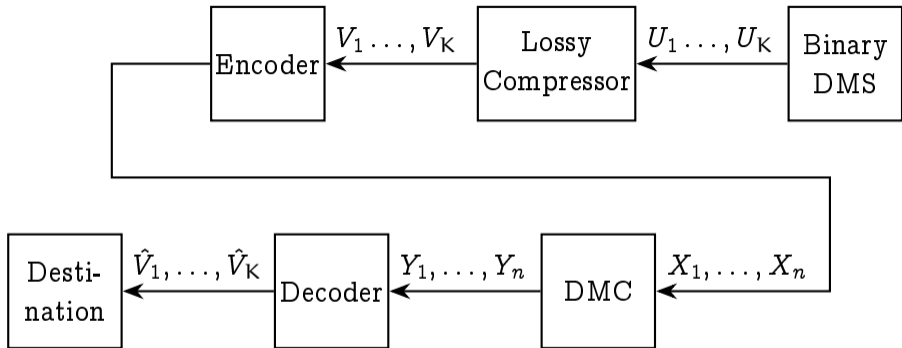


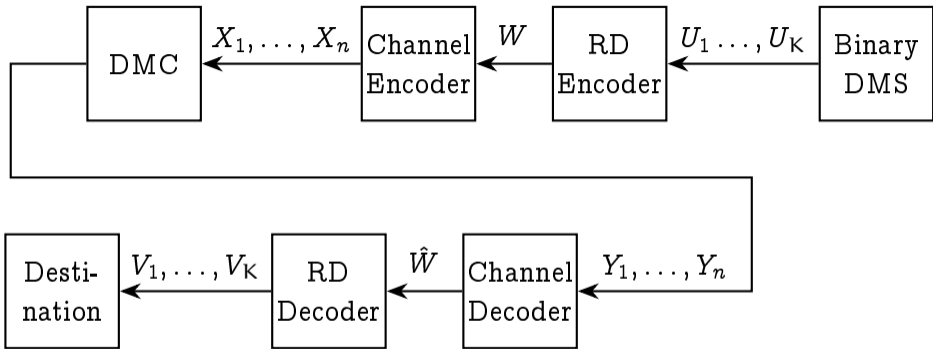




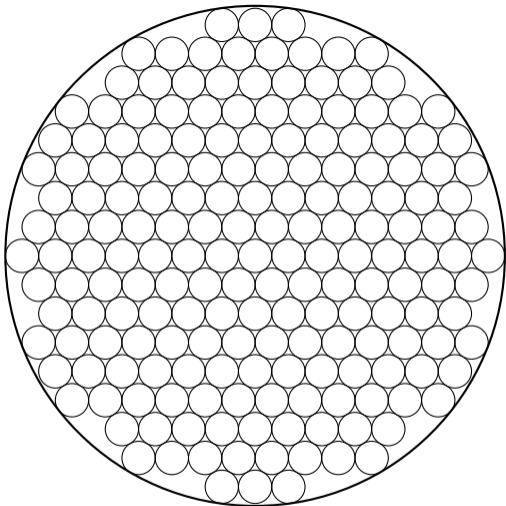


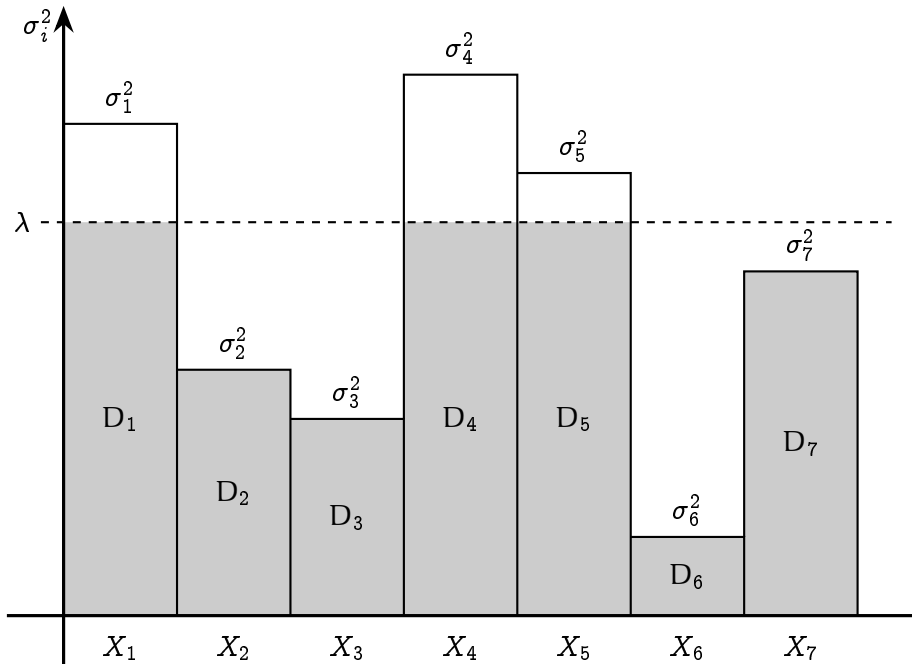
Stefan M. Moser, *Advanced Topics in Information Theory*, Version 5.7.

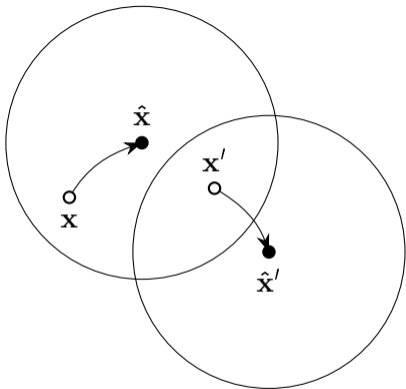






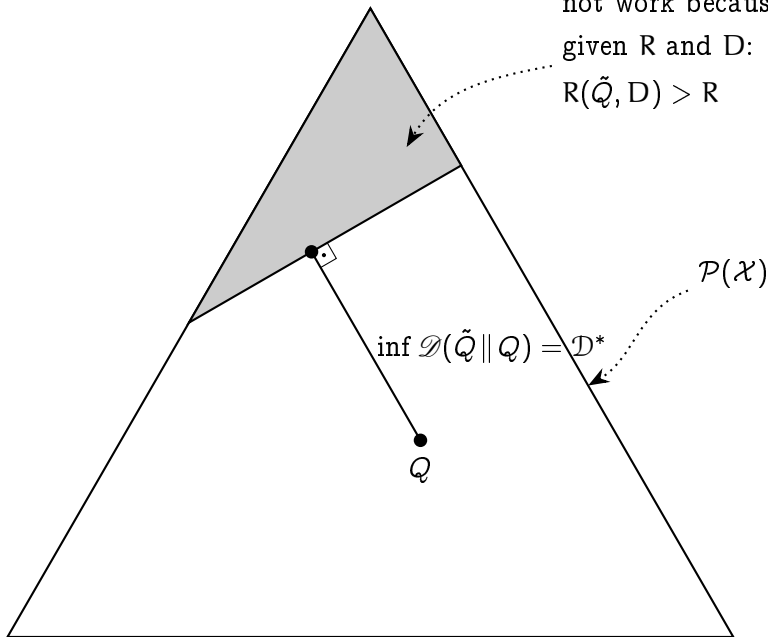


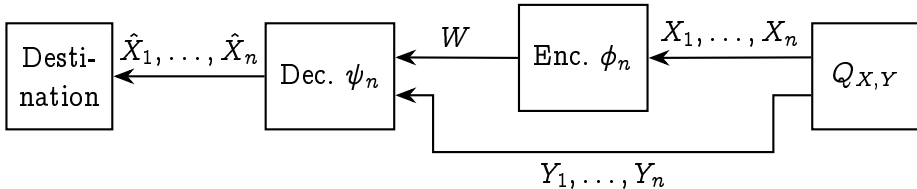


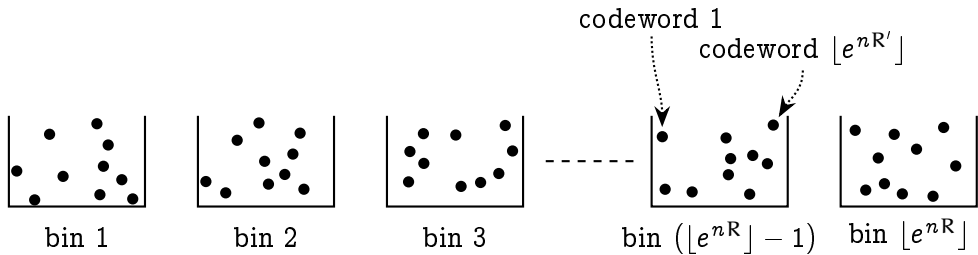


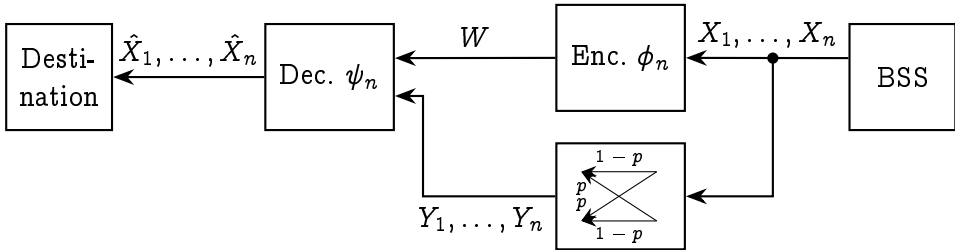
the sources  $\tilde{Q}$  that do not work because for the given  $R$  and  $D$ :

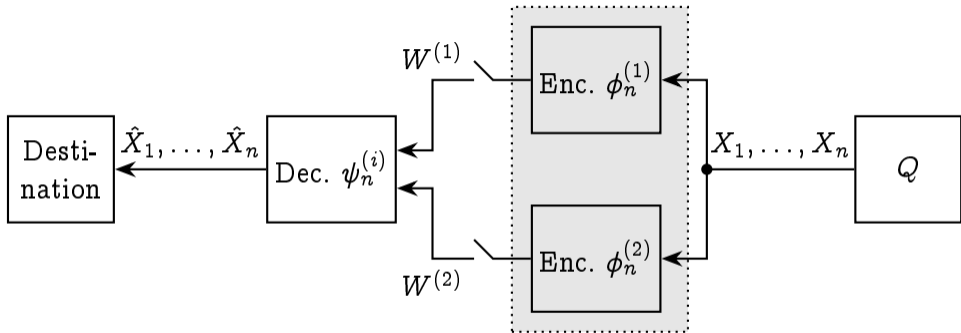
$$R(\tilde{Q}, D) > R$$







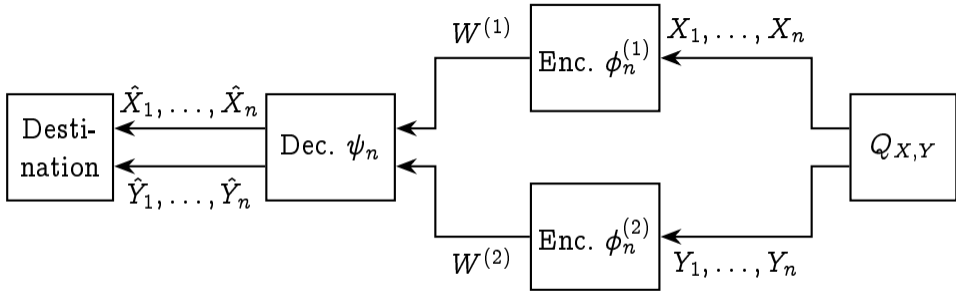


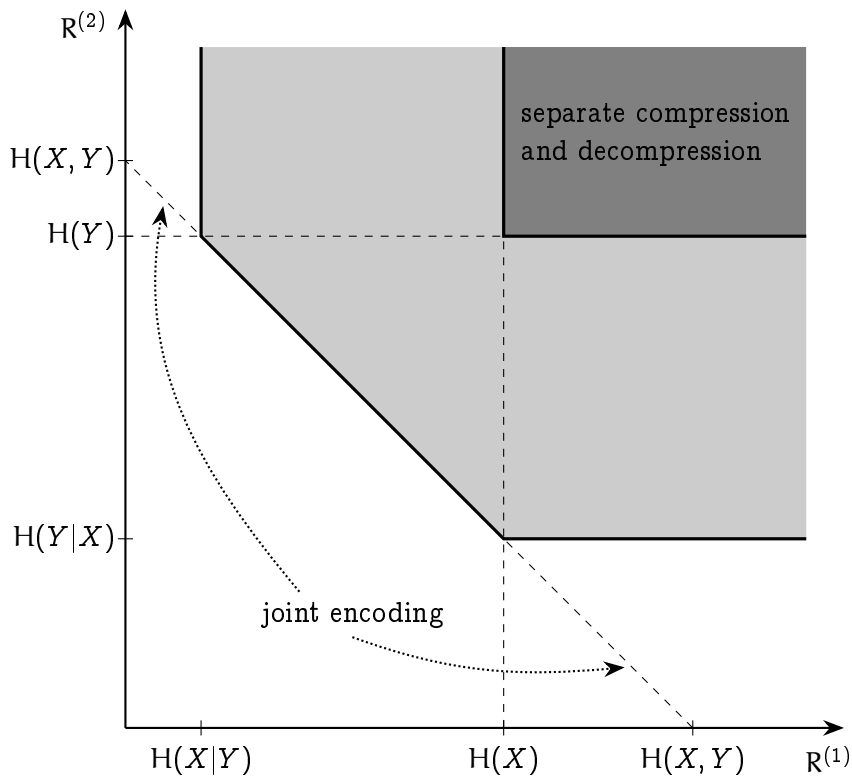




$Q_{\hat{X}^{(1)}, \hat{X}^{(2)} X}(\cdot, \cdot 0)$	$\hat{X}^{(2)} = 0$	$\hat{X}^{(2)} = 1$	$Q_{\hat{X}^{(1)} X}(\cdot 0)$
$\hat{X}^{(1)} = 0$	$3 - 2\sqrt{2}$	$\sqrt{2} - 1$	$2 - \sqrt{2}$
$\hat{X}^{(1)} = 1$	$\sqrt{2} - 1$	0	$\sqrt{2} - 1$
$Q_{\hat{X}^{(2)} X}(\cdot 0)$	$2 - \sqrt{2}$	$\sqrt{2} - 1$	
$Q_{\hat{X}^{(1)}, \hat{X}^{(2)} X}(\cdot, \cdot 1)$	$\hat{X}^{(2)} = 0$	$\hat{X}^{(2)} = 1$	$Q_{\hat{X}^{(1)} X}(\cdot 1)$
$\hat{X}^{(1)} = 0$	0	0	0
$\hat{X}^{(1)} = 1$	0	1	1
$Q_{\hat{X}^{(2)} X}(\cdot 1)$	0	1	

$Q_{\hat{X}^{(1)}, \hat{X}^{(2)}}(\cdot, \cdot)$	$\hat{X}^{(2)} = 0$	$\hat{X}^{(2)} = 1$	$Q_{\hat{X}^{(1)}}(\cdot)$
$\hat{X}^{(1)} = 0$	$\frac{3}{2} - \sqrt{2}$	$\frac{\sqrt{2}}{2} - \frac{1}{2}$	$1 - \frac{\sqrt{2}}{2}$
$\hat{X}^{(1)} = 1$	$\frac{\sqrt{2}}{2} - \frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$
$Q_{\hat{X}^{(2)}}(\cdot)$	$1 - \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	

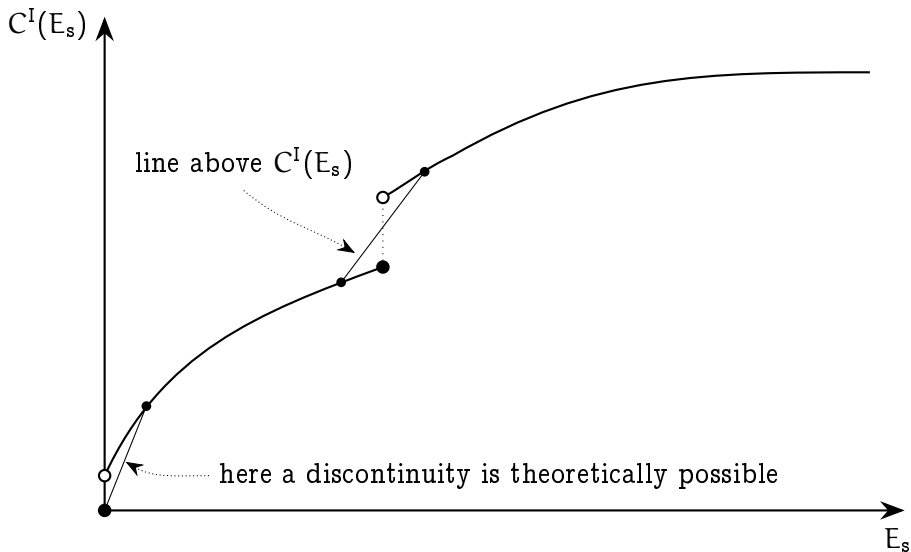


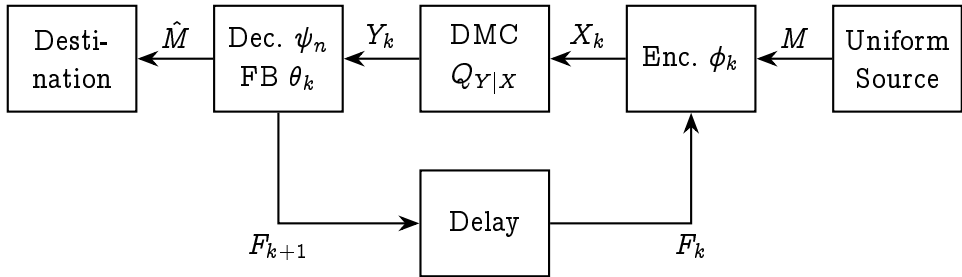


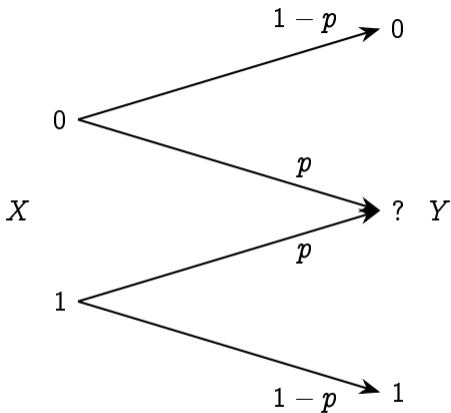
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$Q_{X,Y}(\cdot, \cdot)$		Taichung $Y$		
		rain	sun	Hsinchu total
Hsinchu $X$	rain	0.445	0.055	0.5
	sun	0.055	0.445	0.5
Taichung total		0.5	0.5	

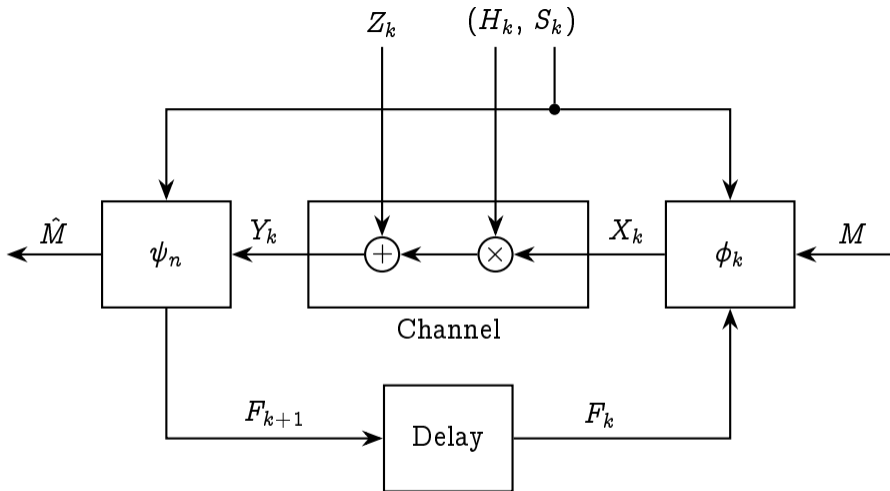
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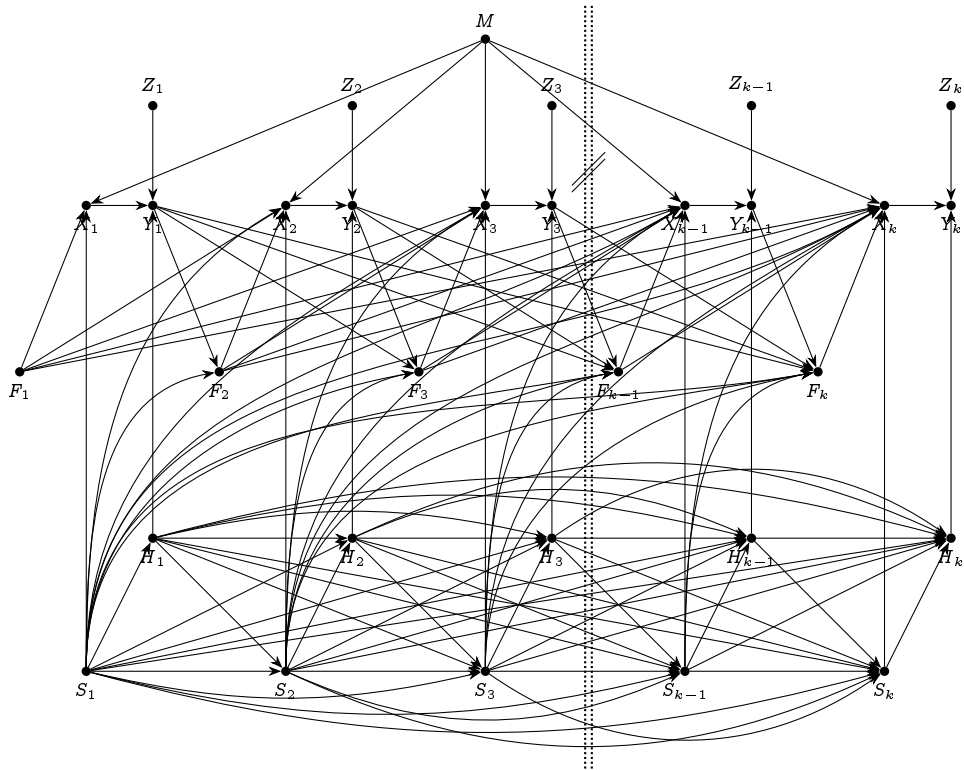


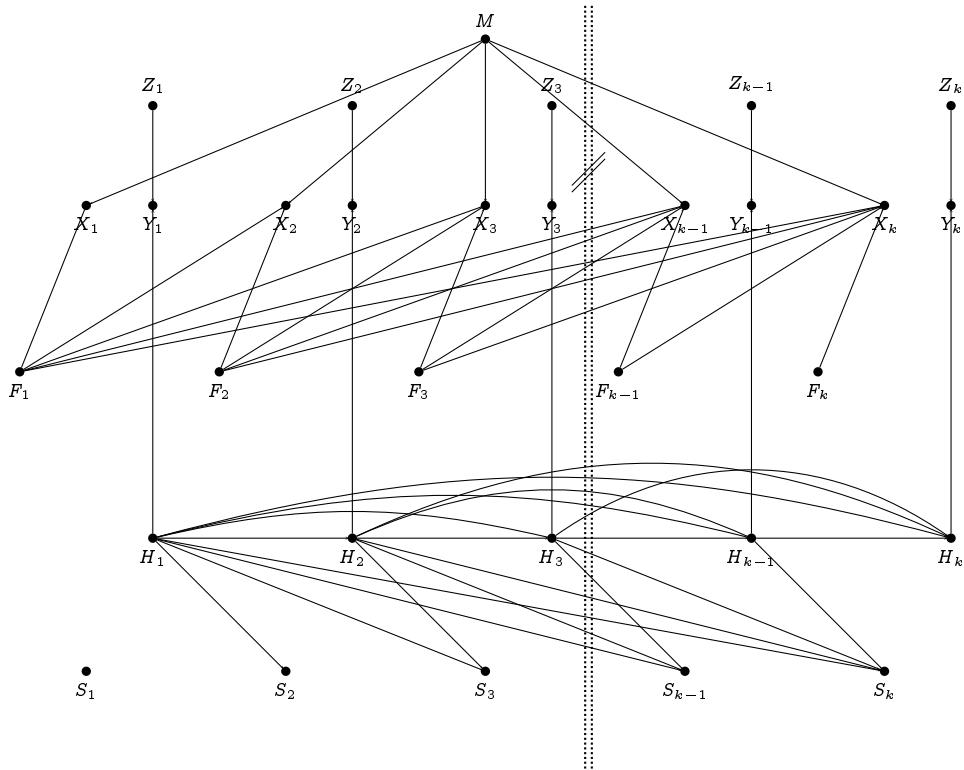


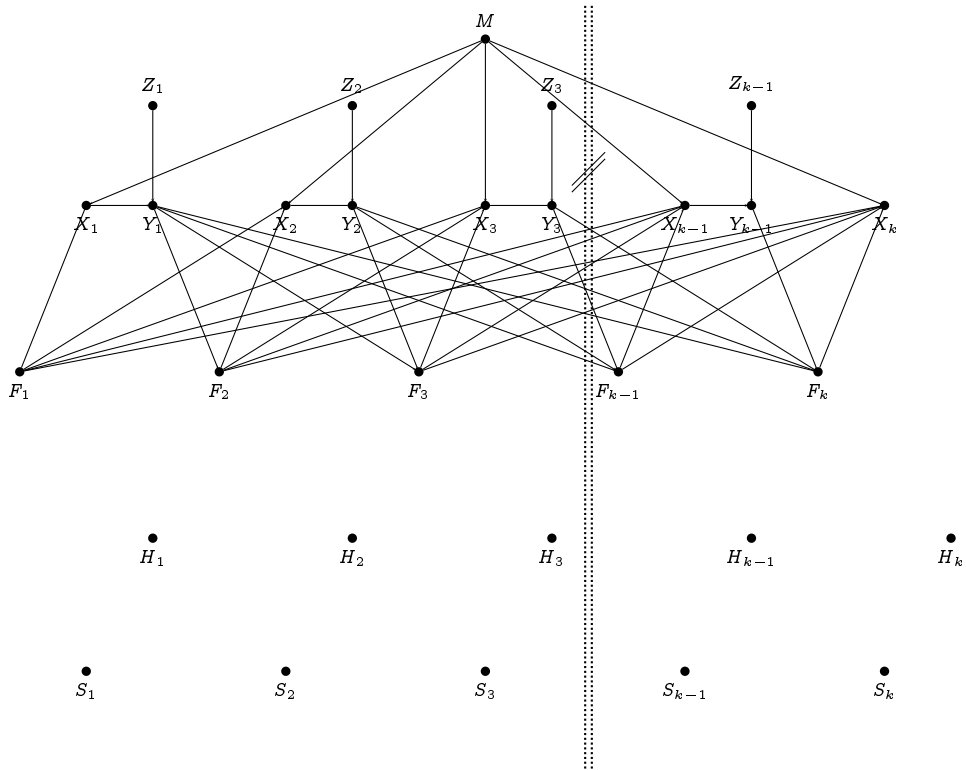


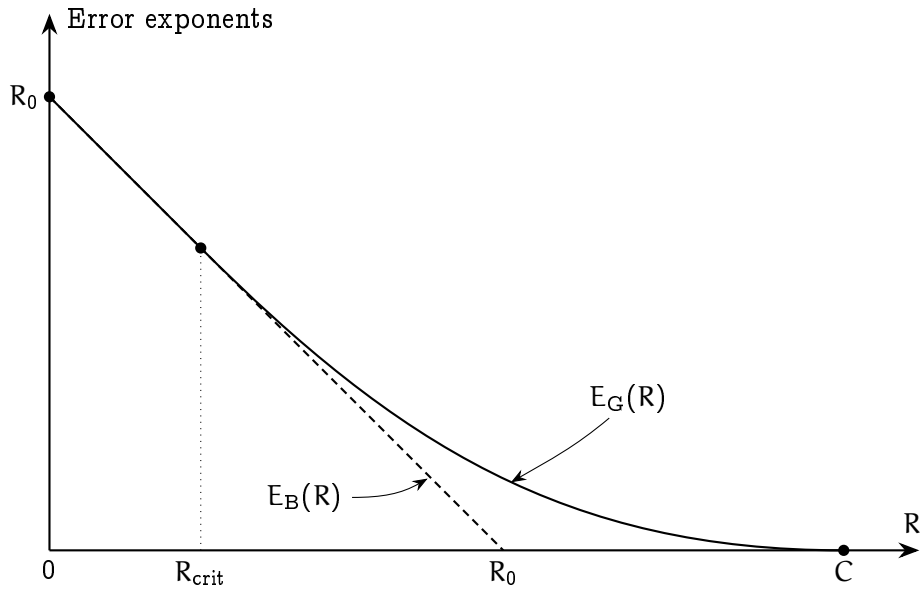


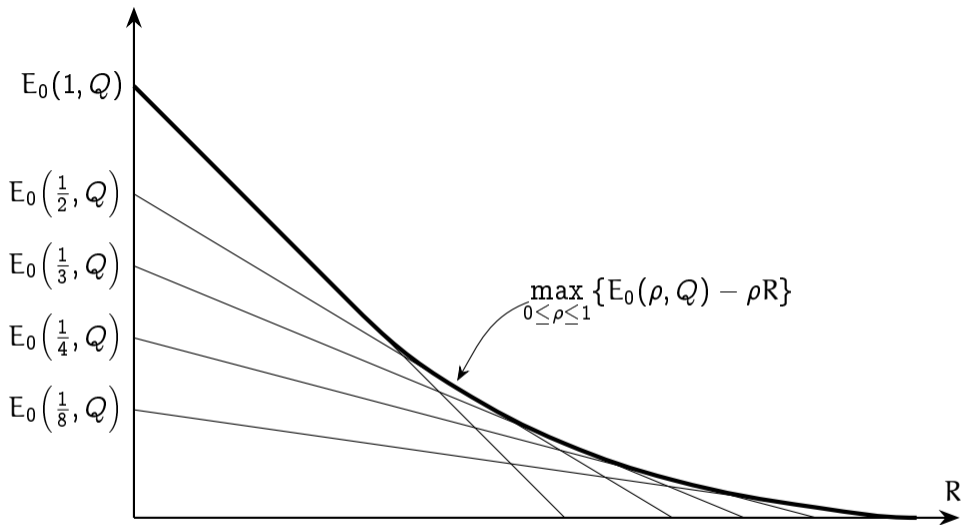


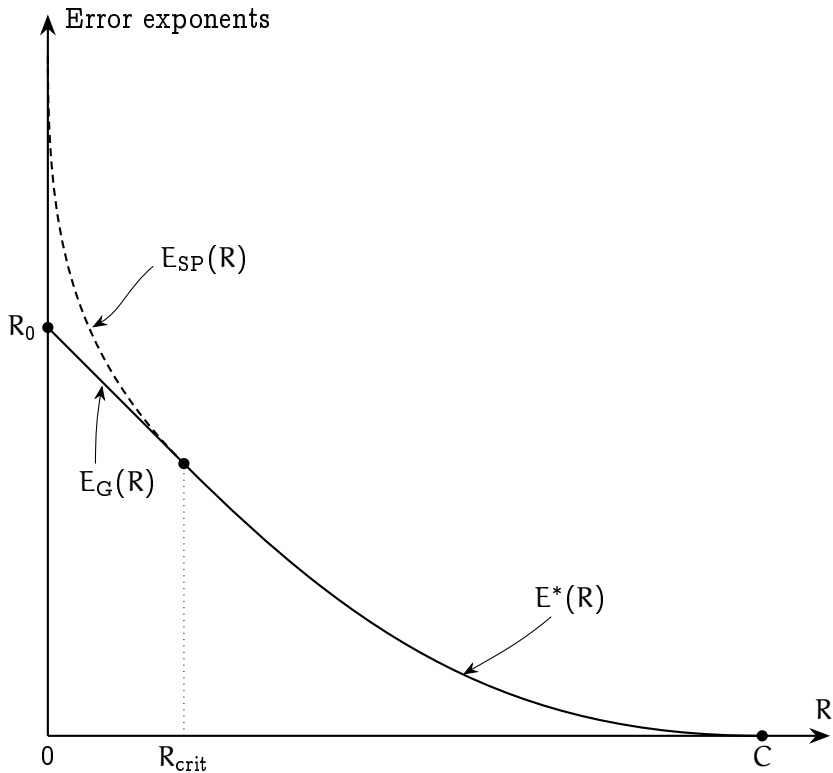


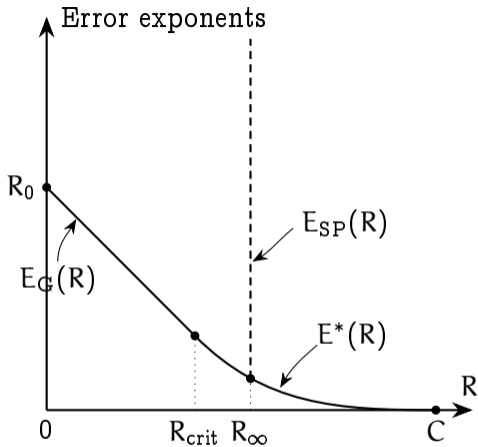
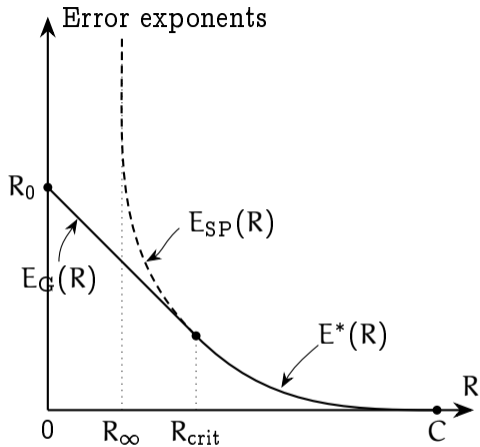




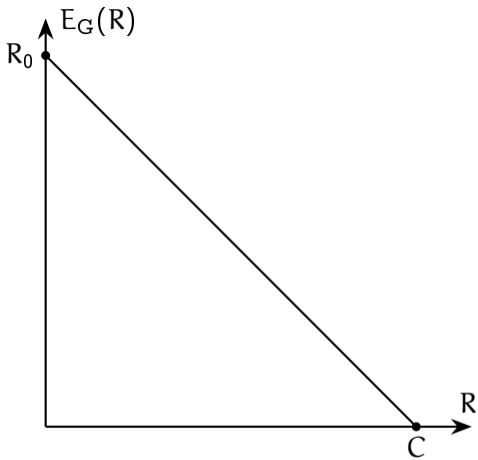
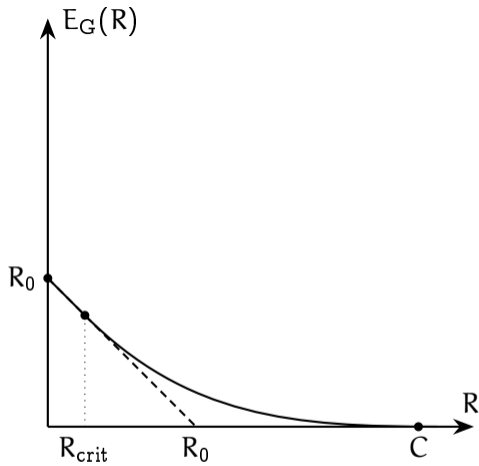


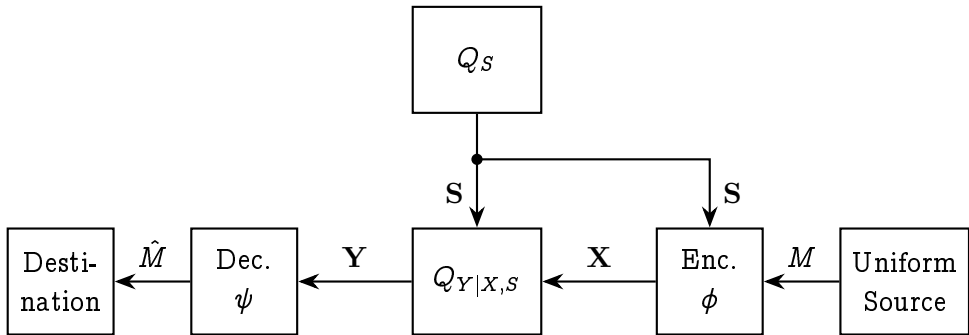


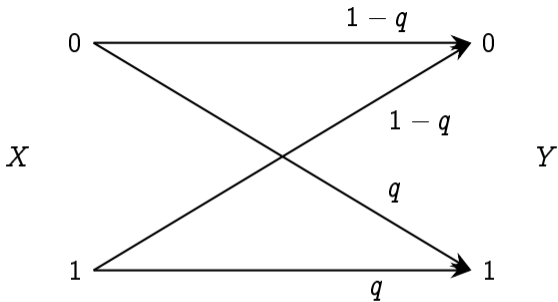


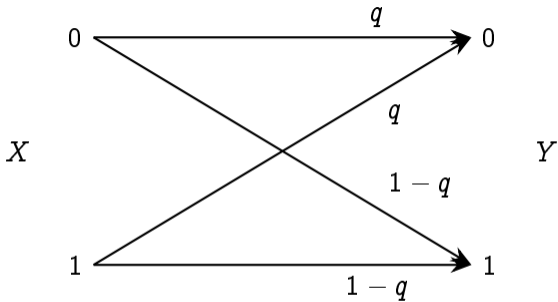


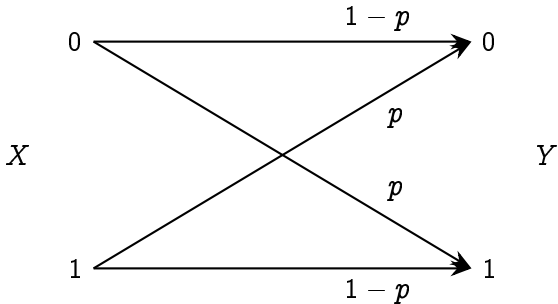


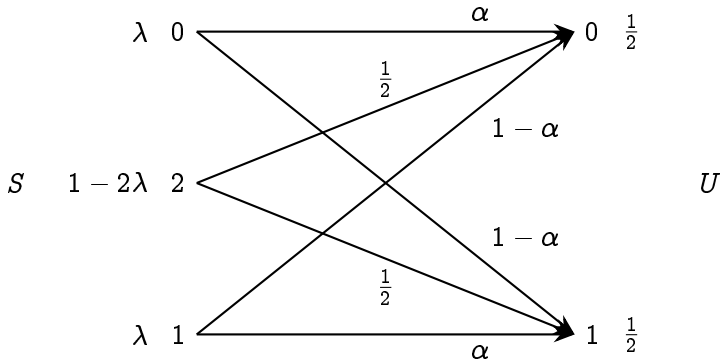


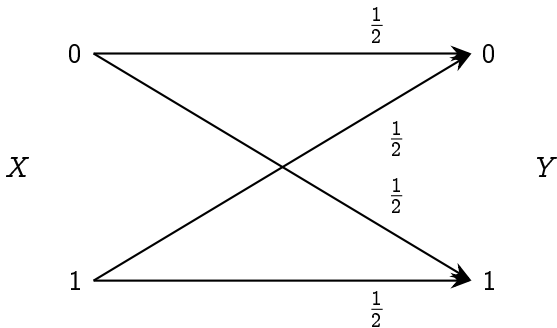


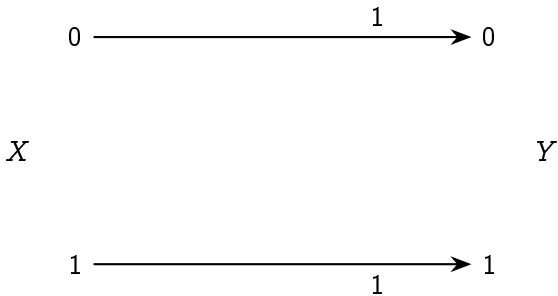




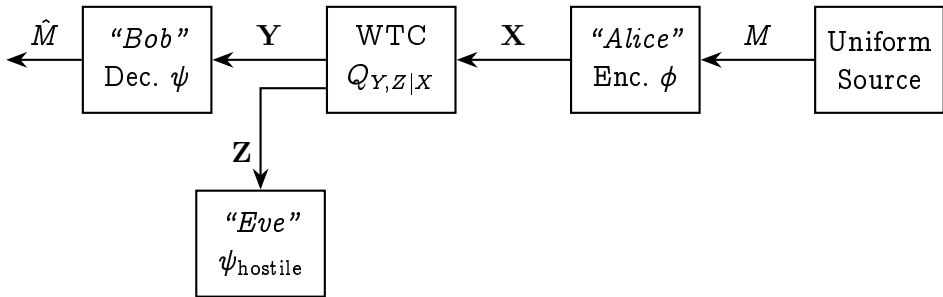


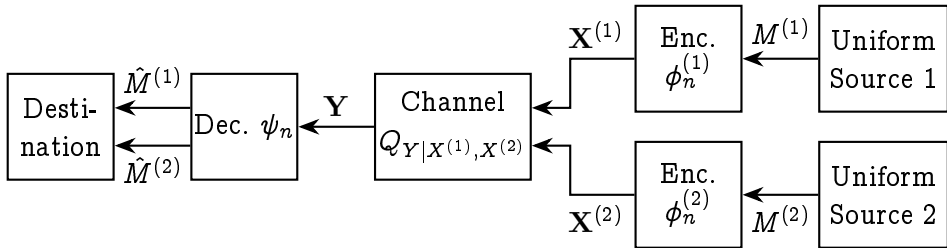


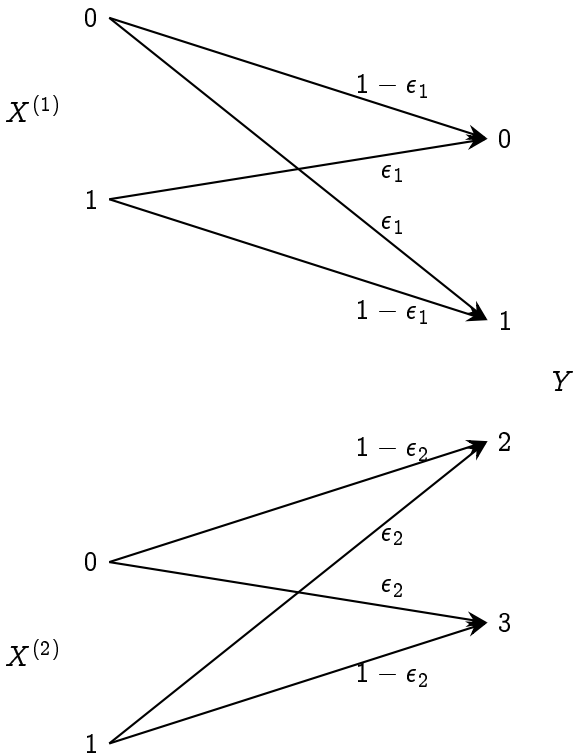


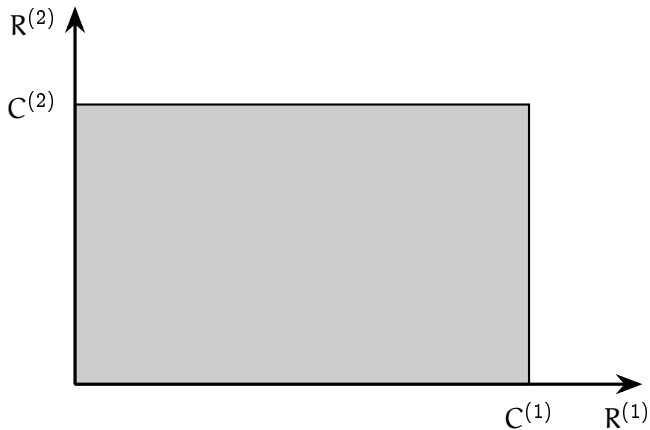


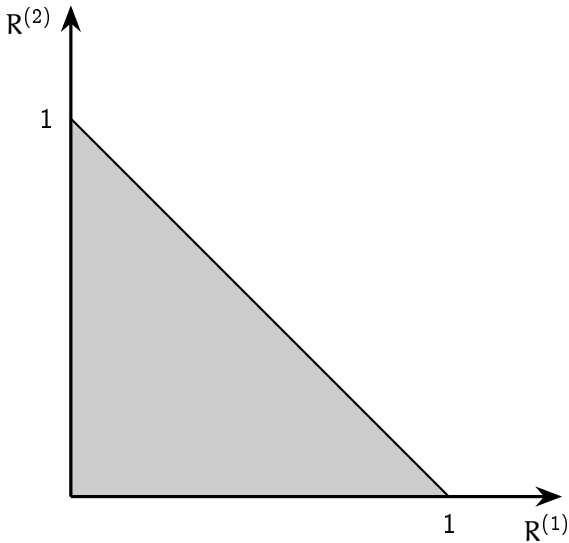


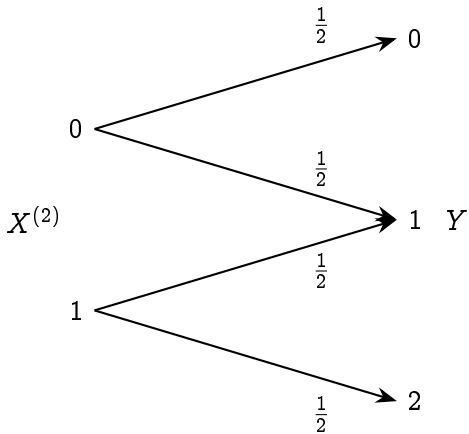


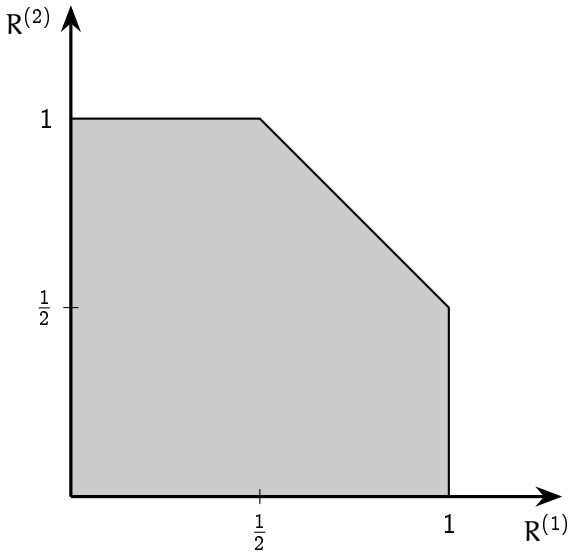


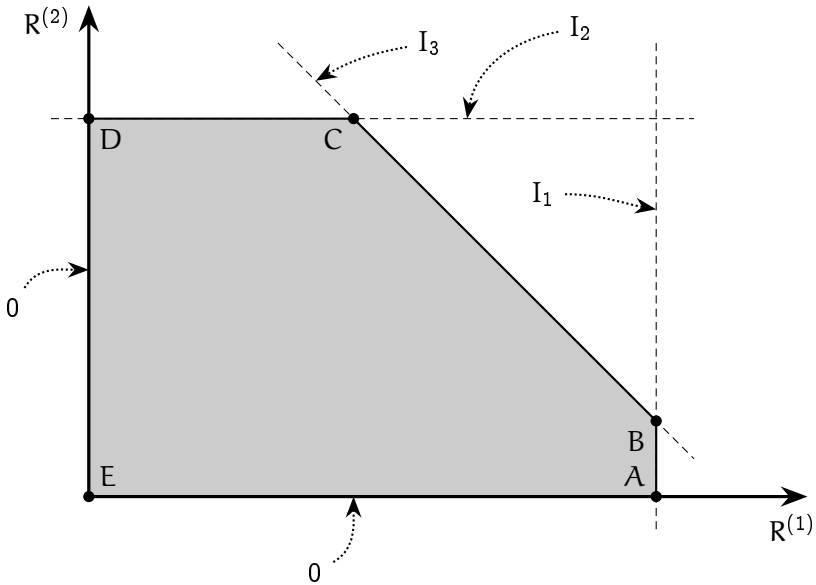




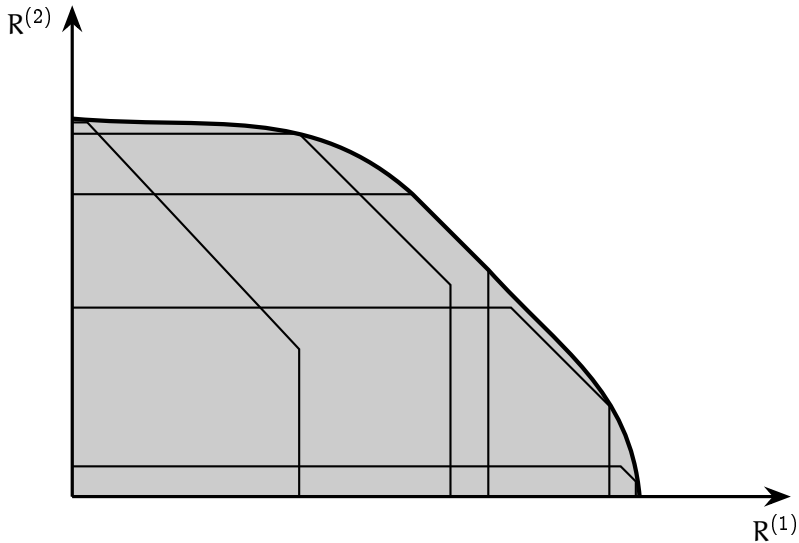


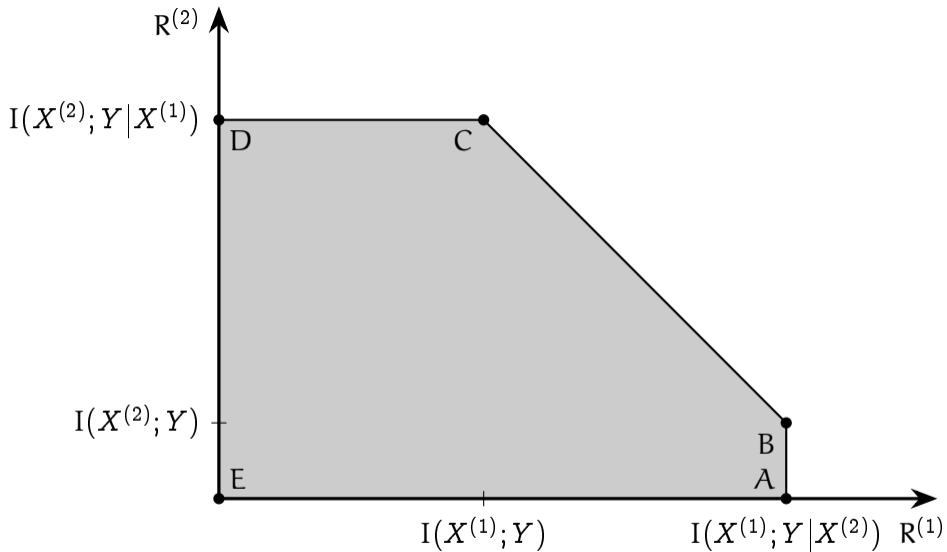


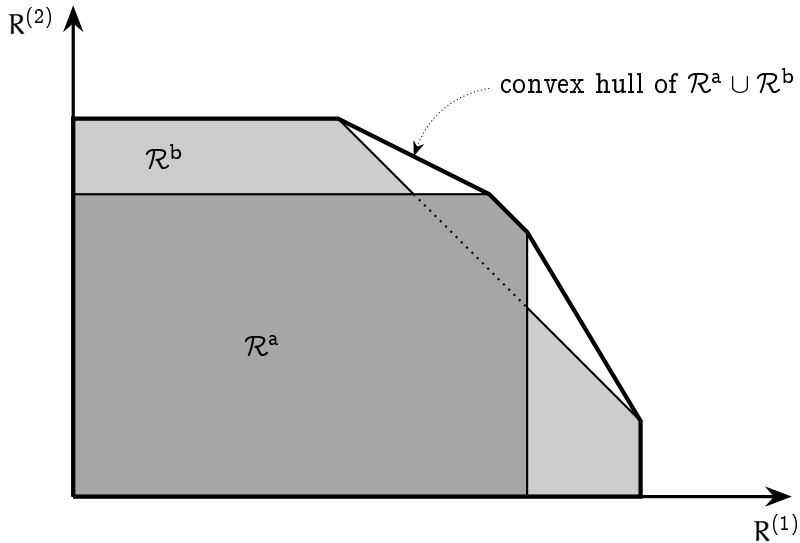


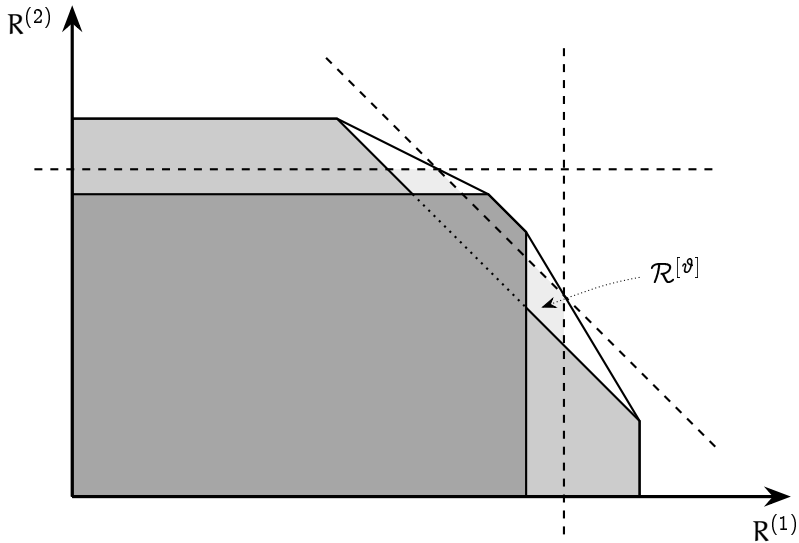


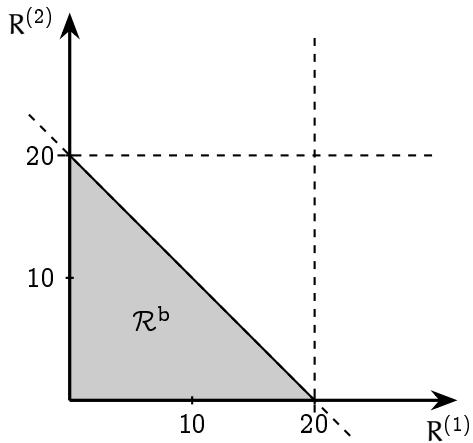
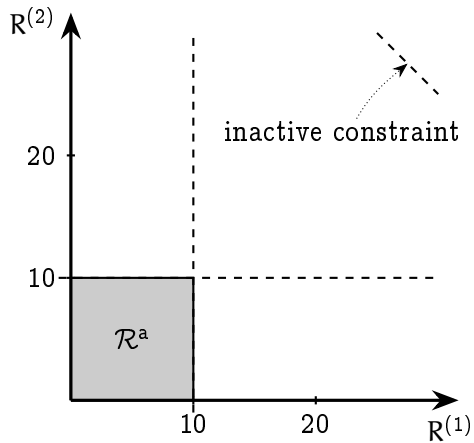


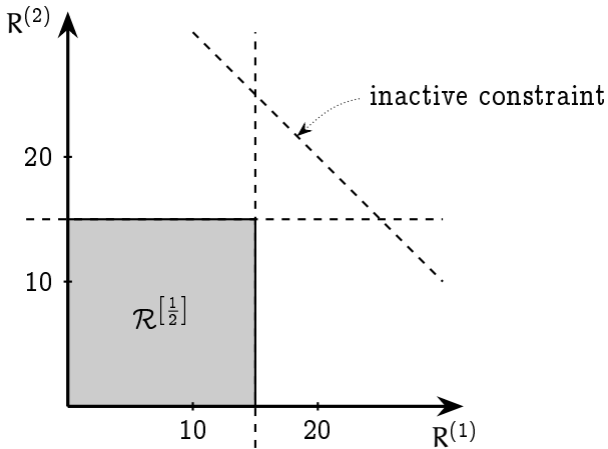


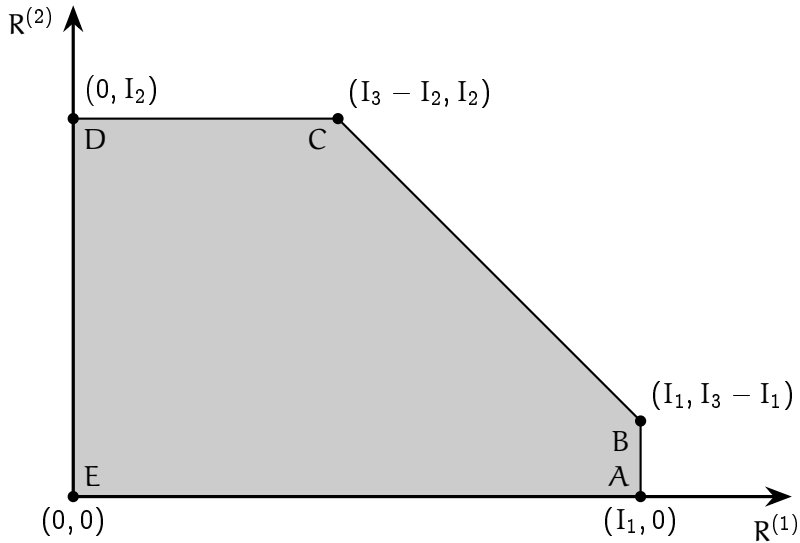


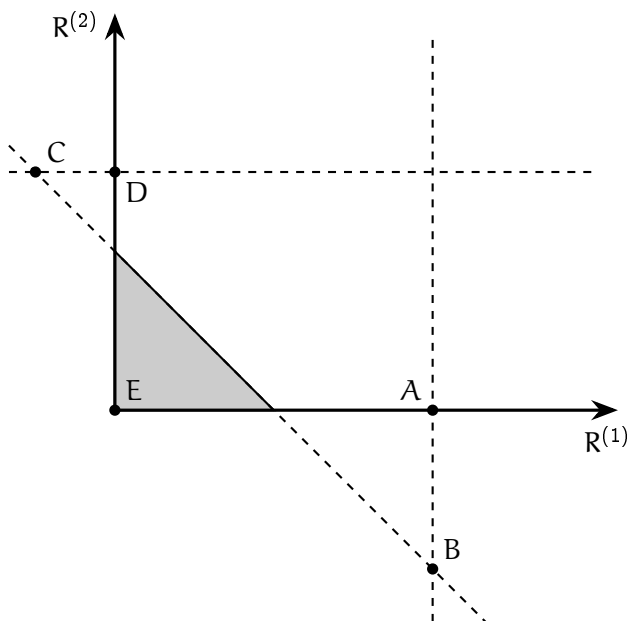
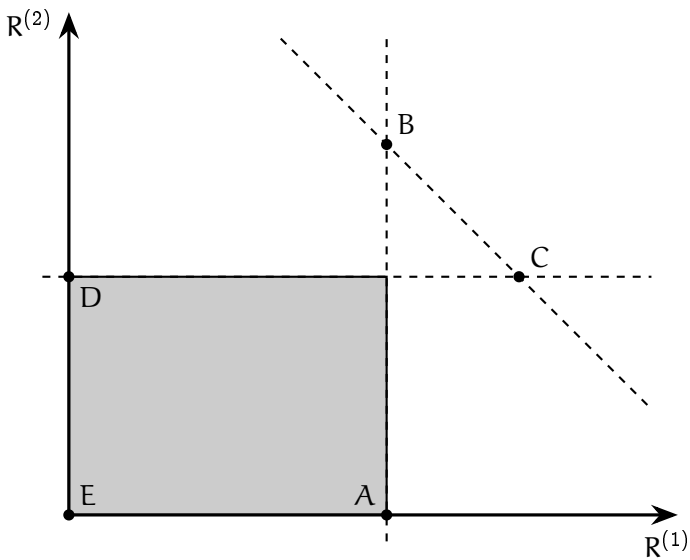




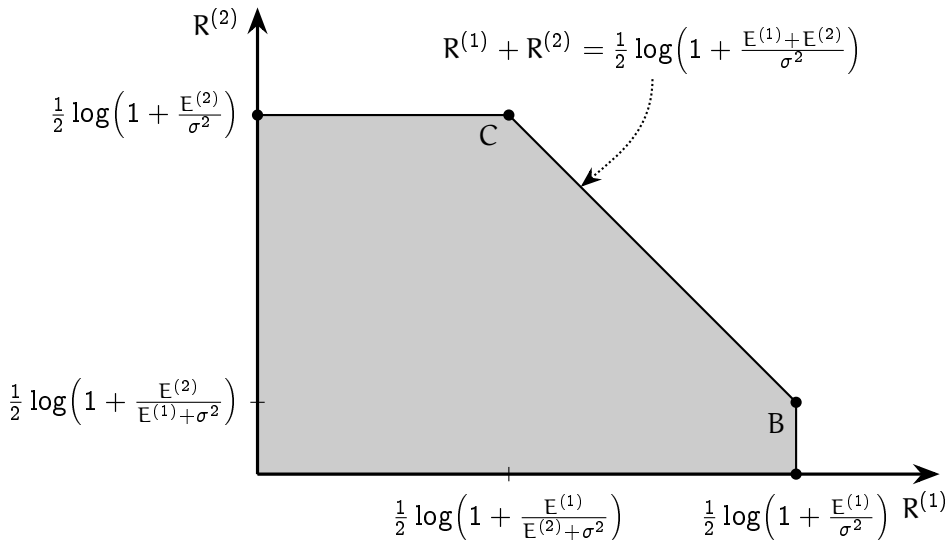


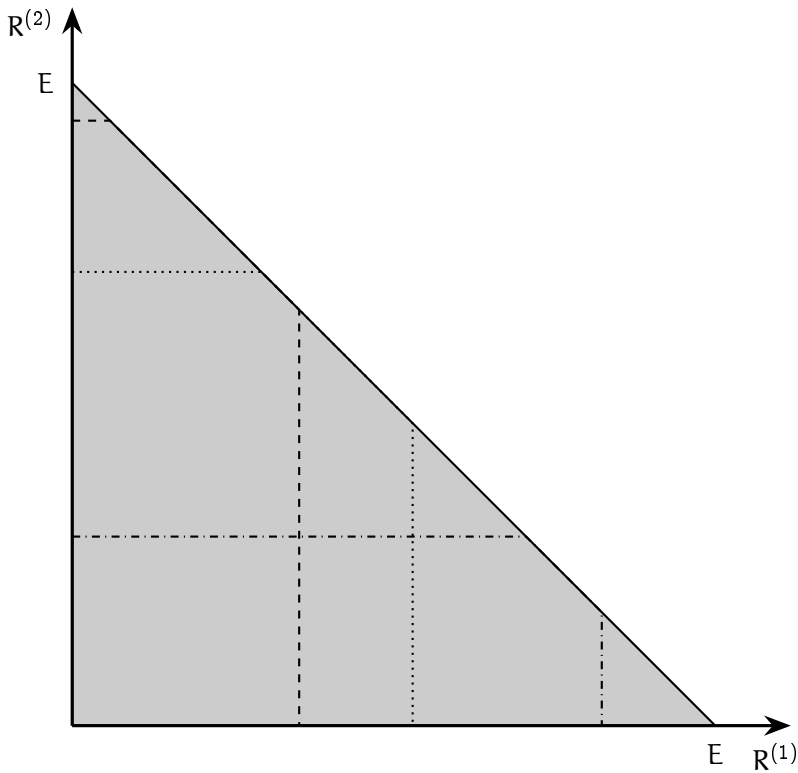


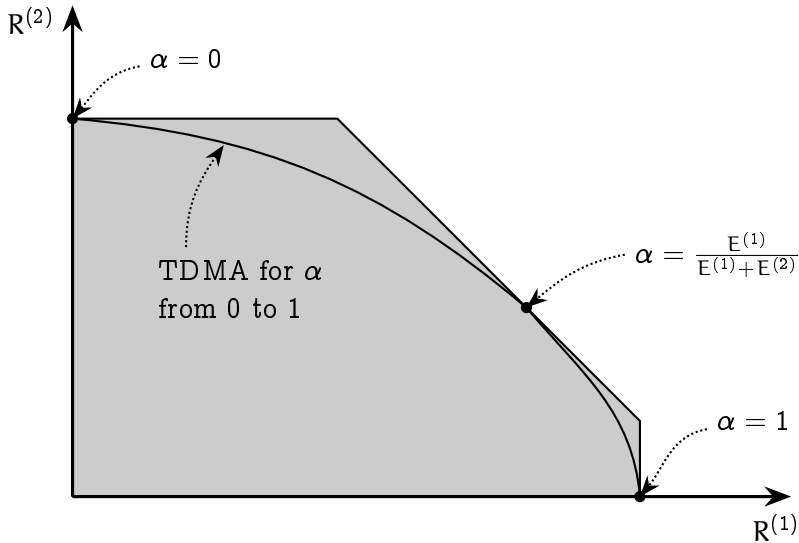


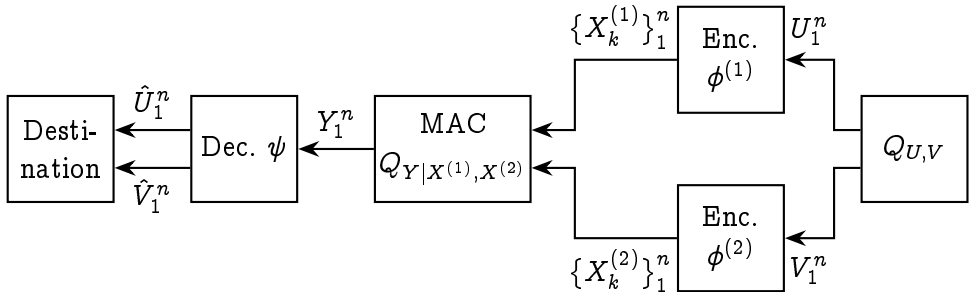


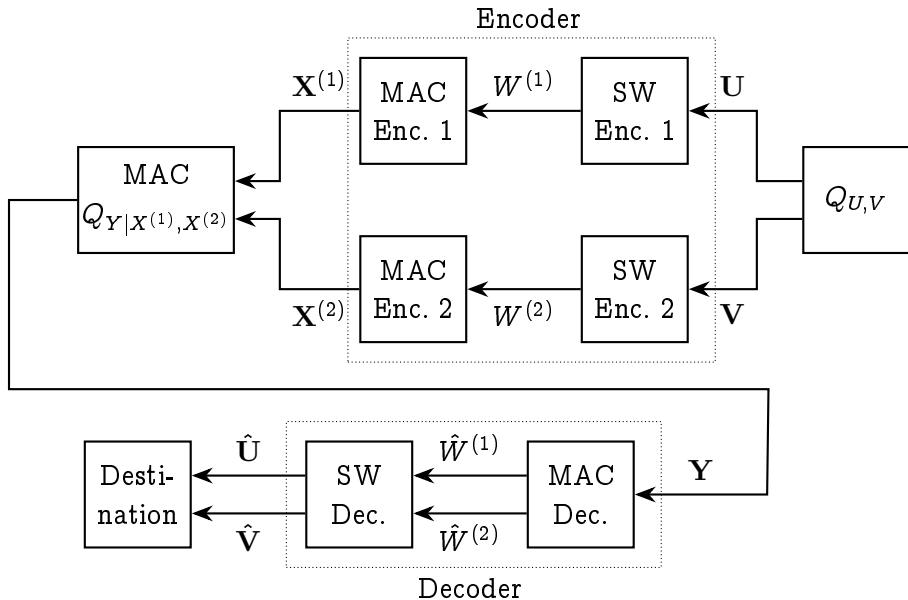


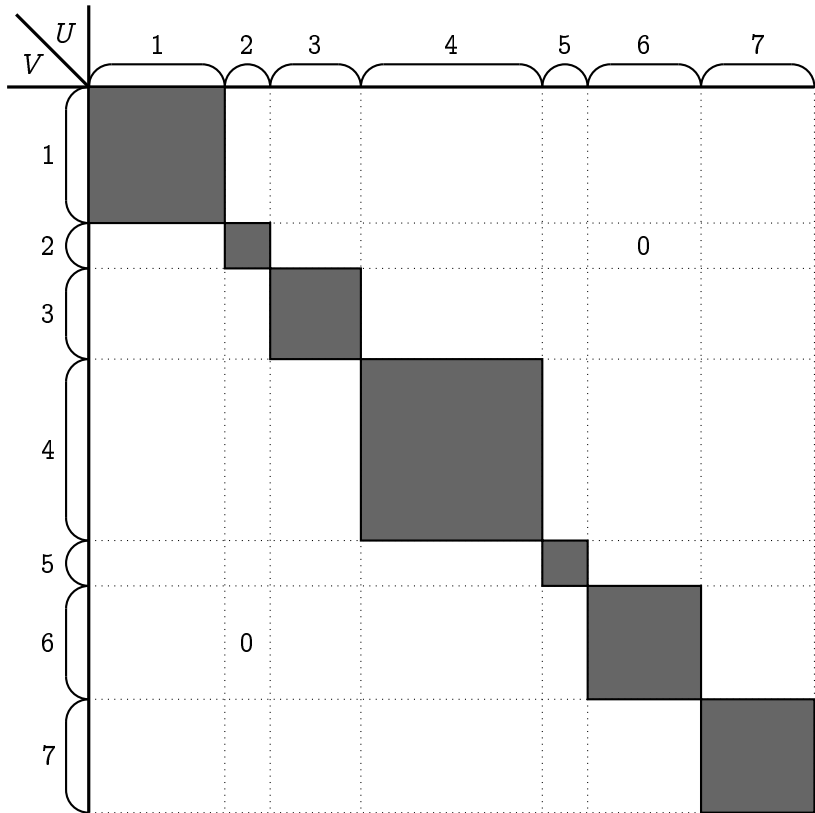


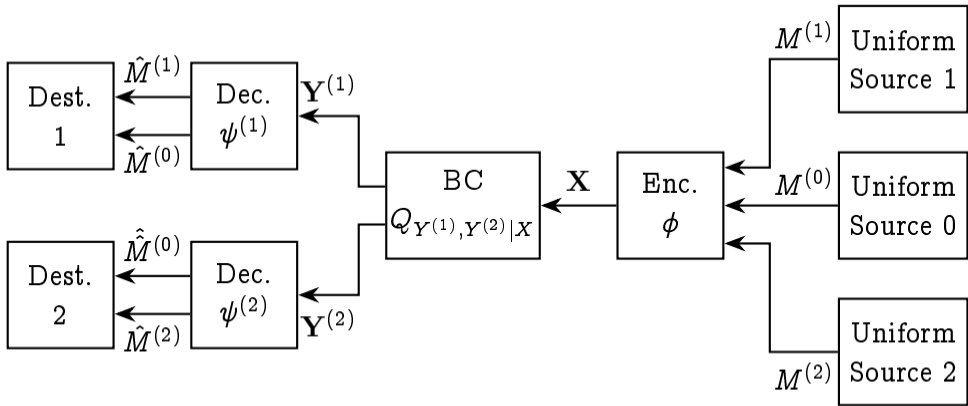


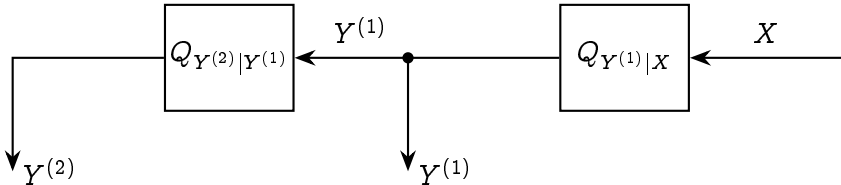




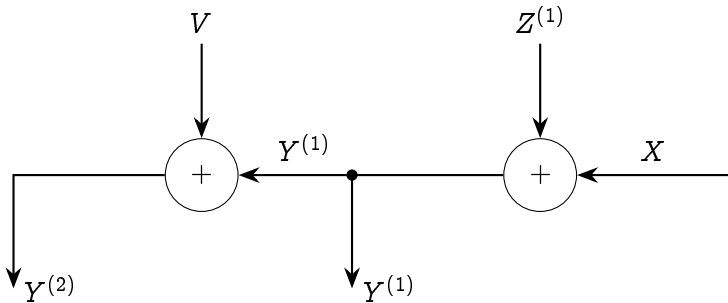


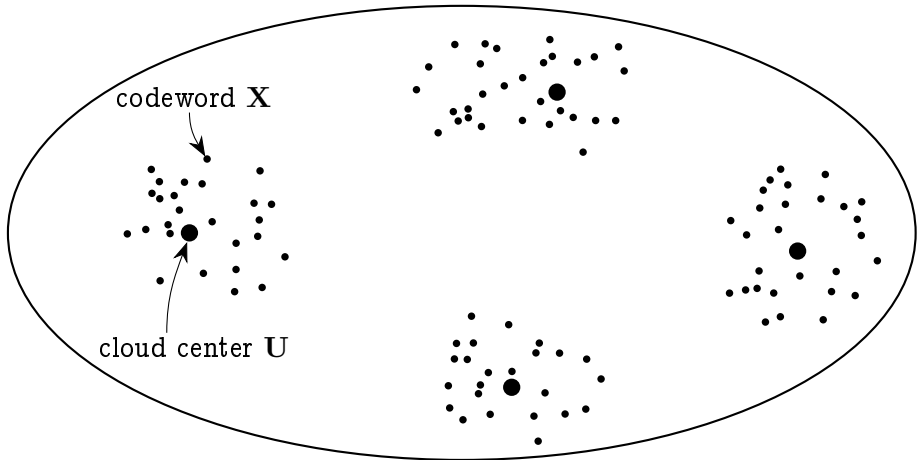


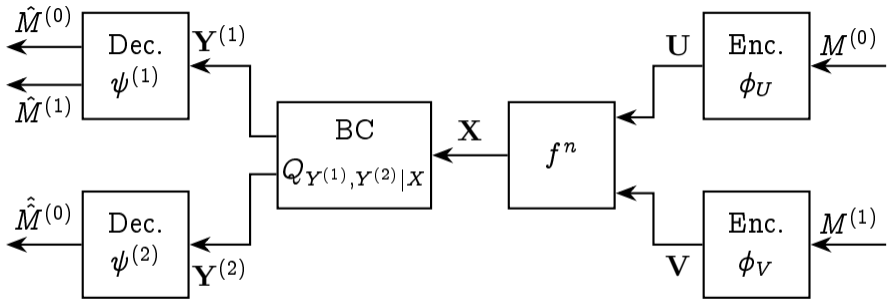


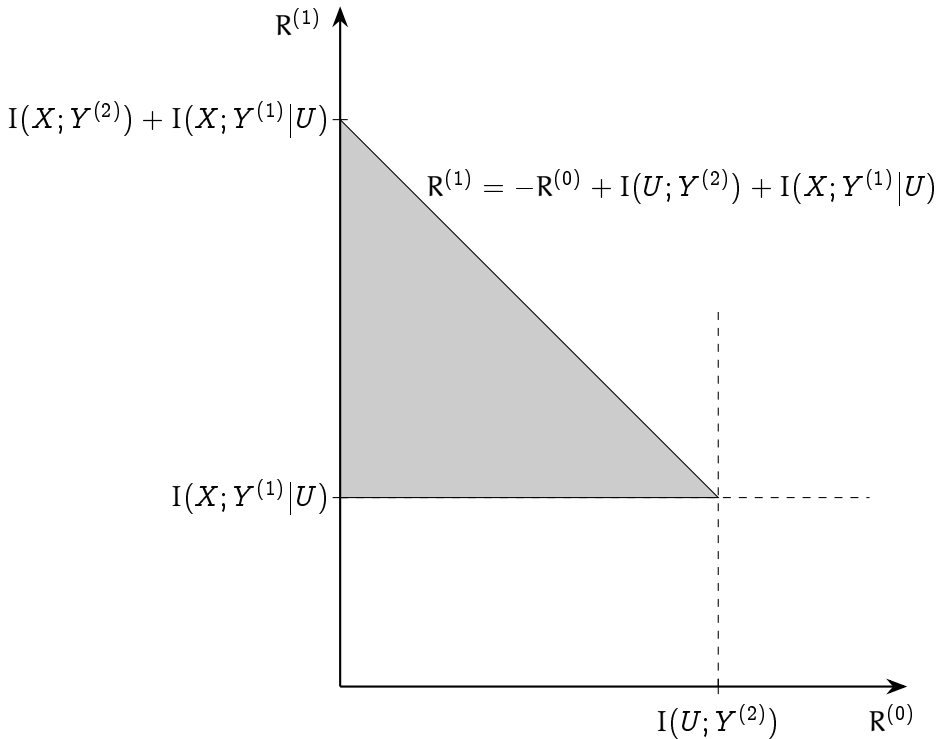




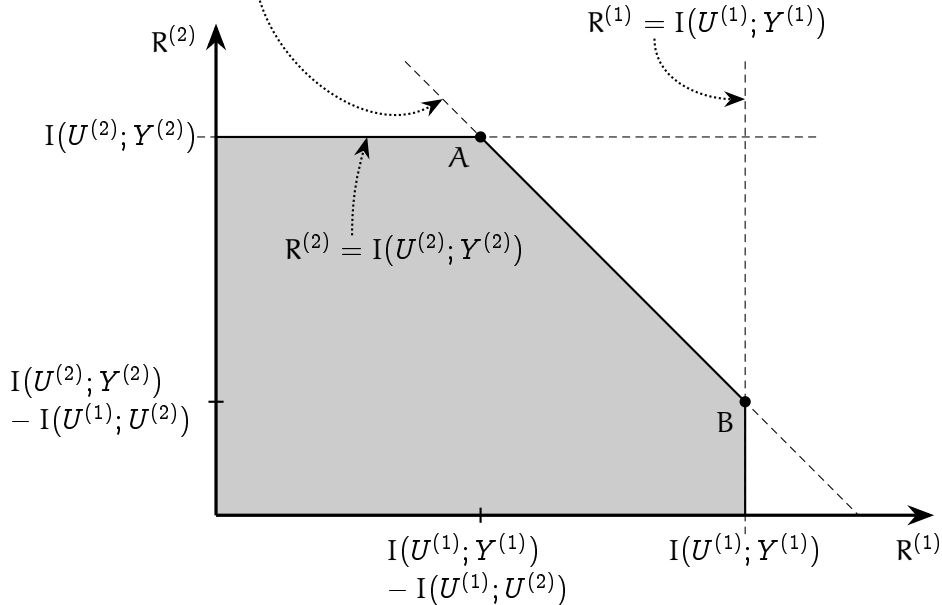


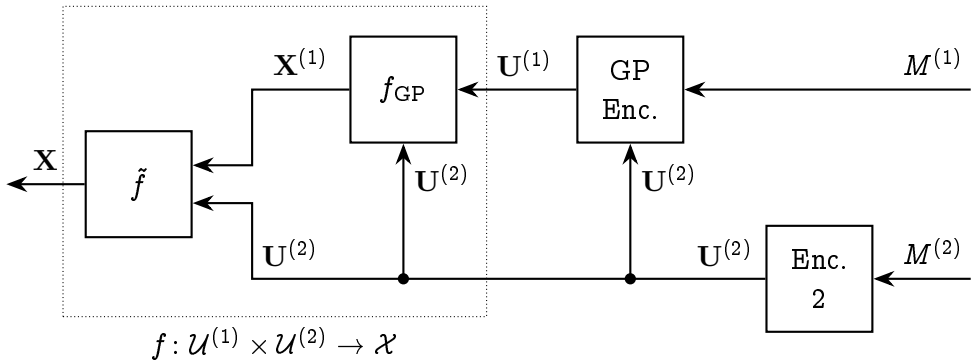


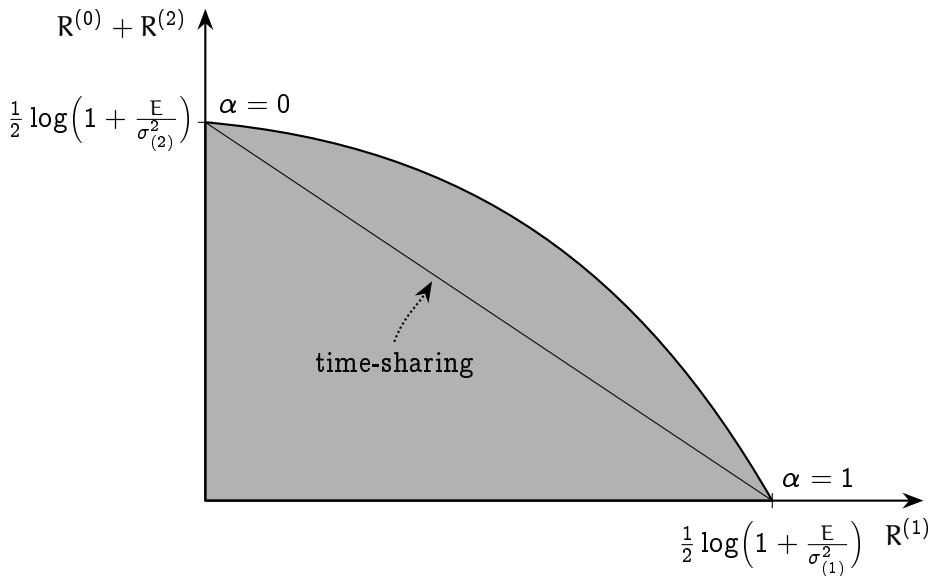


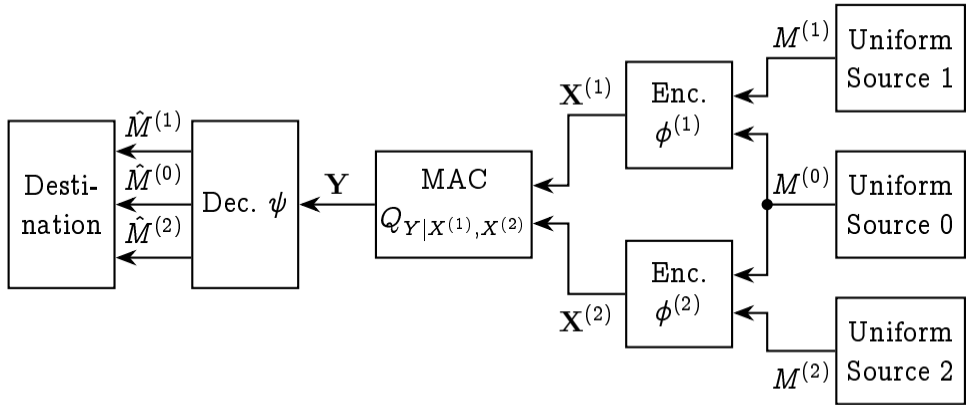


$$R^{(2)} = -R^{(1)} + I(U^{(1)}; Y^{(1)}) + I(U^{(2)}; Y^{(2)}) - I(U^{(1)}; U^{(2)})$$

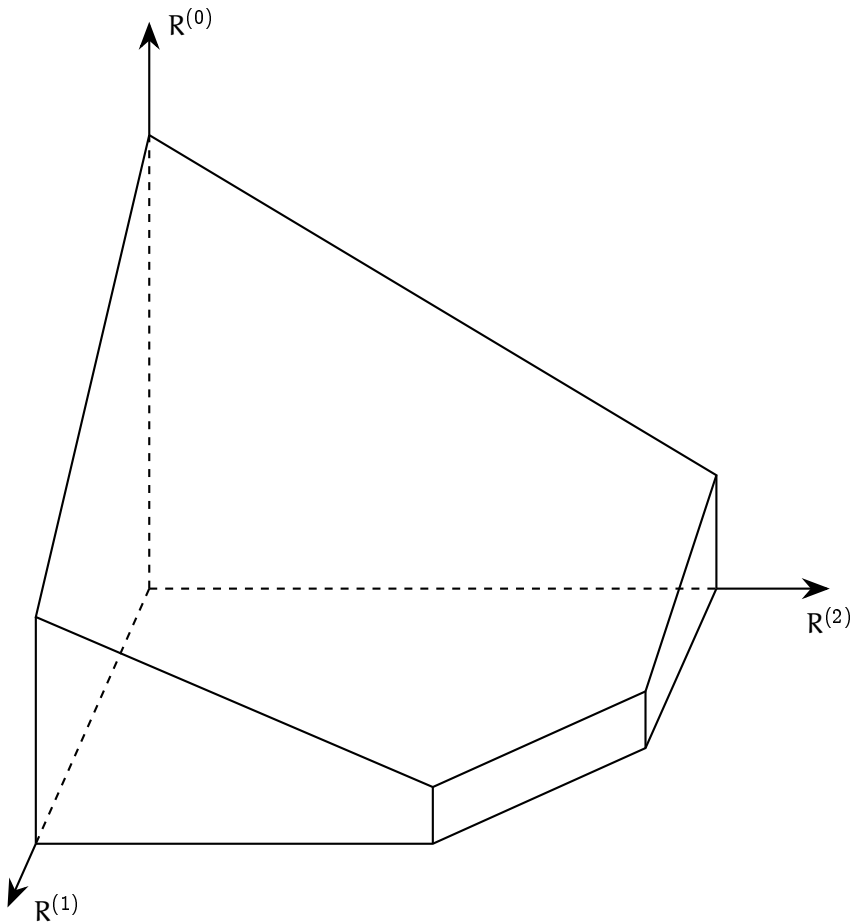


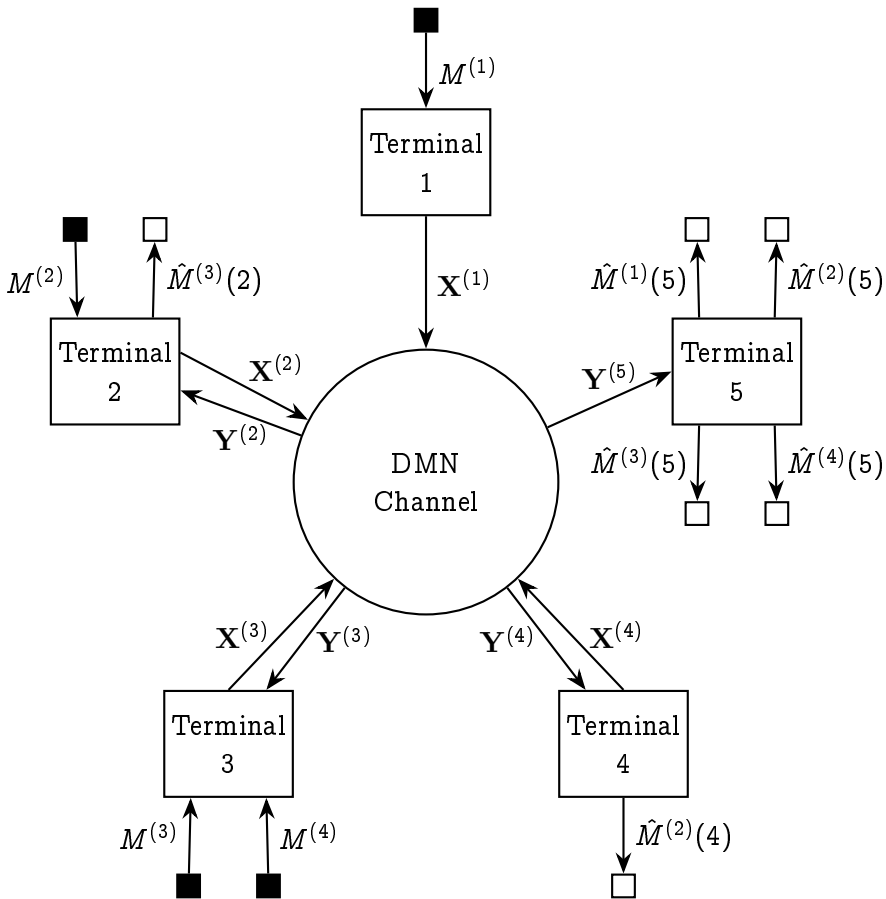


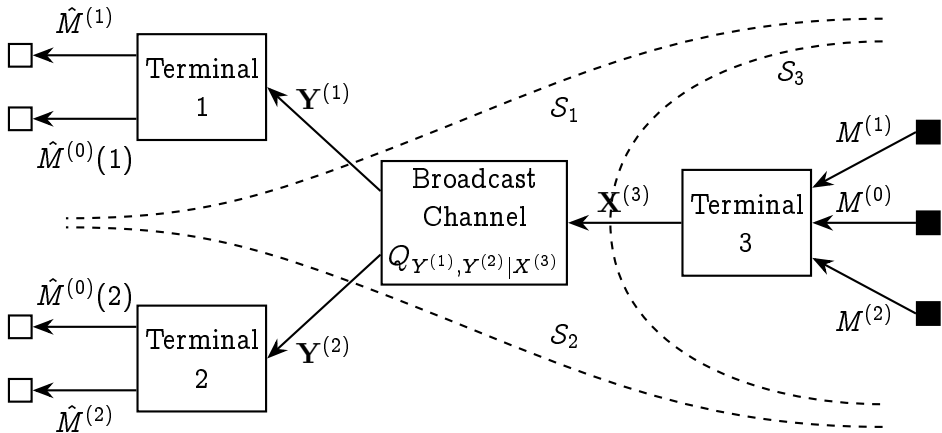


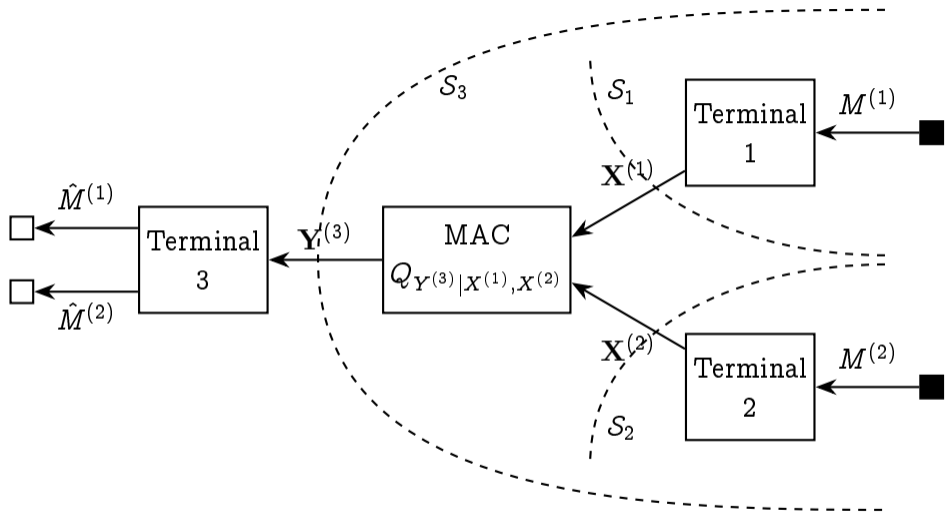


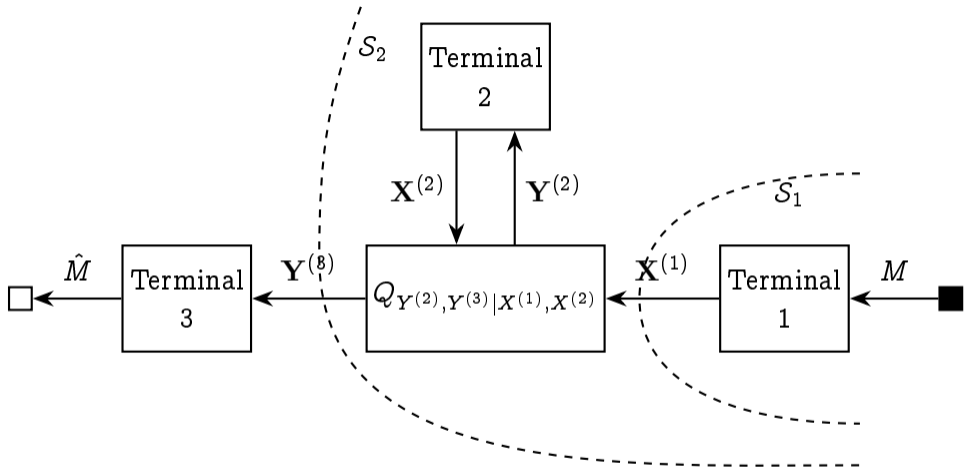


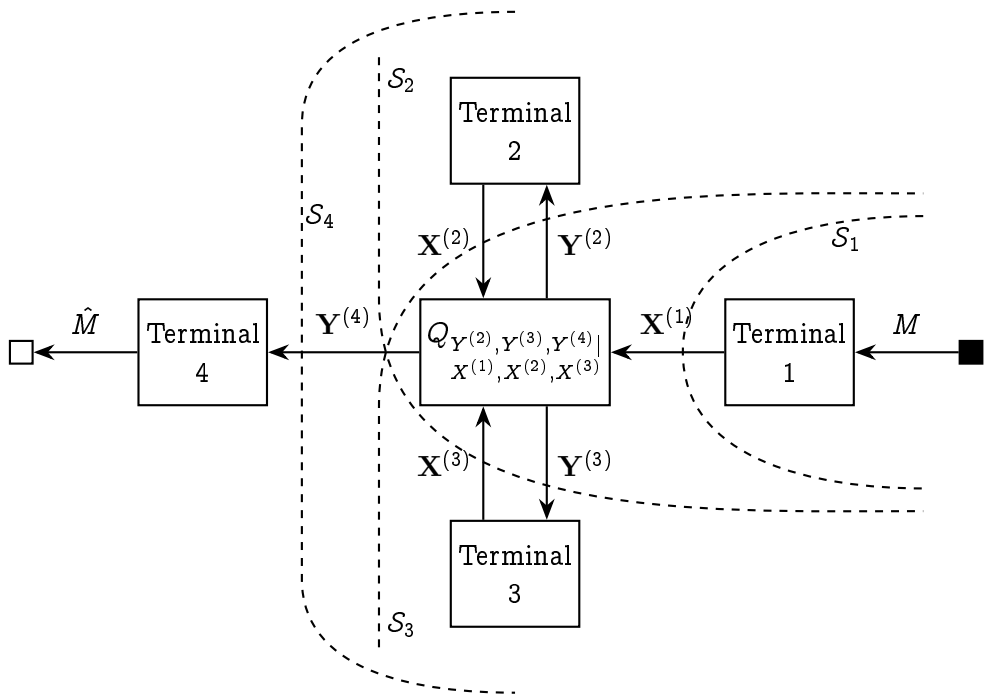


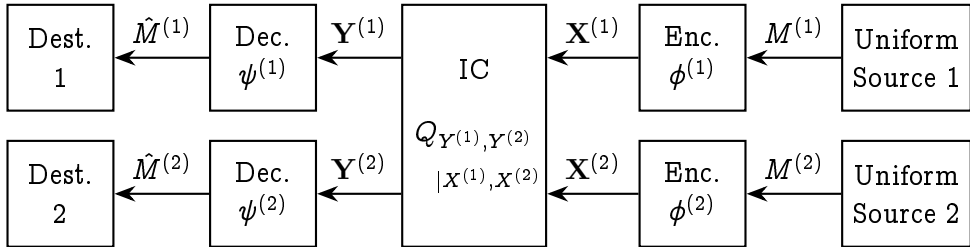


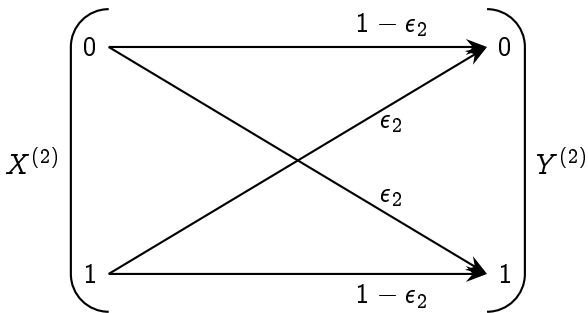
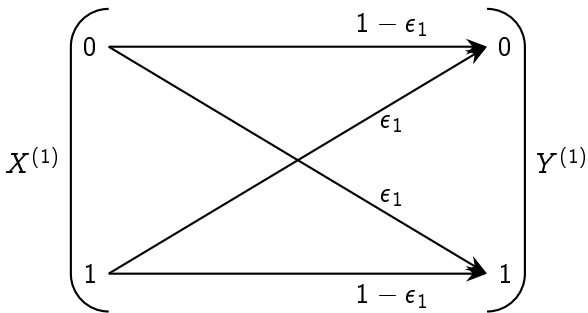




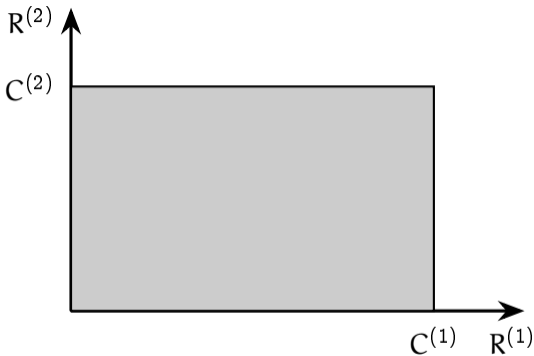


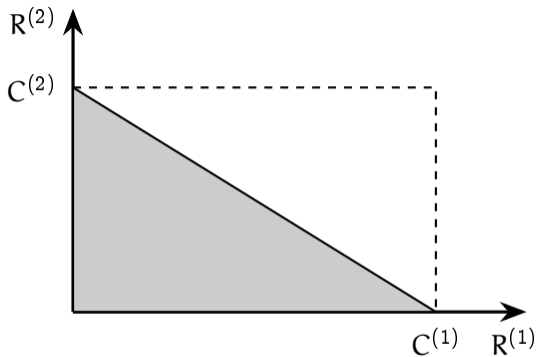


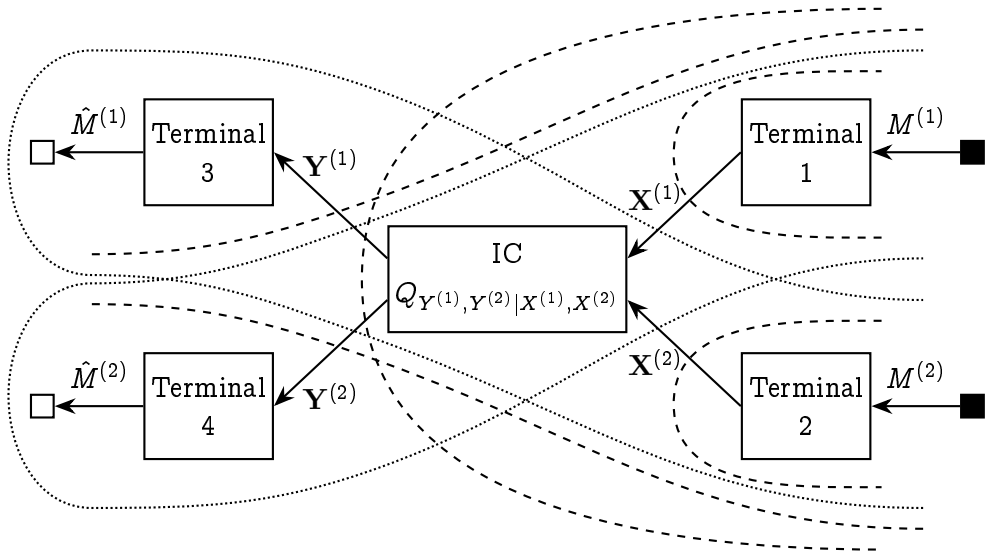




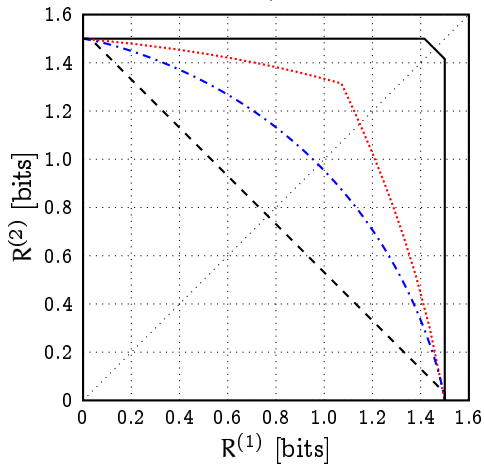




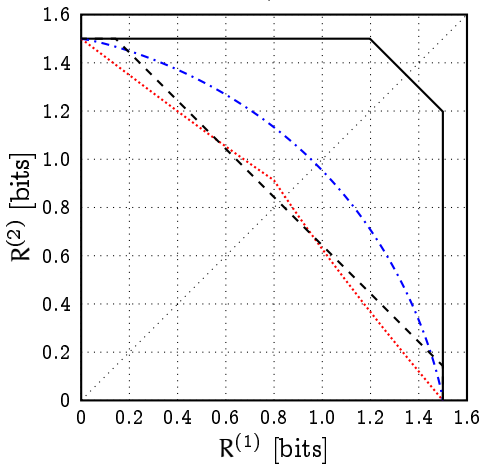




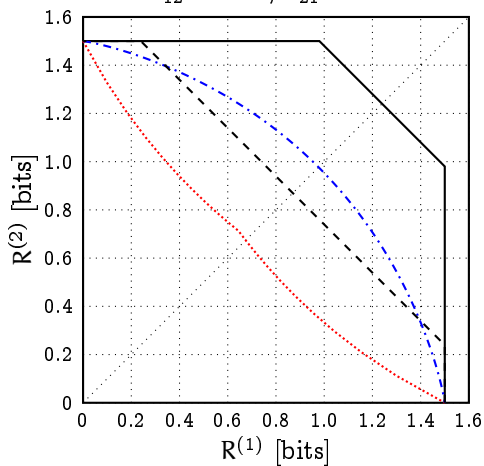
$a_{12} = 0.15, a_{21} = 0.05$



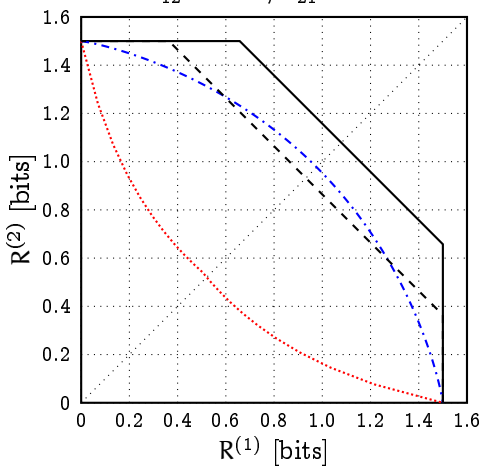
$a_{12} = 0.35, a_{21} = 0.25$



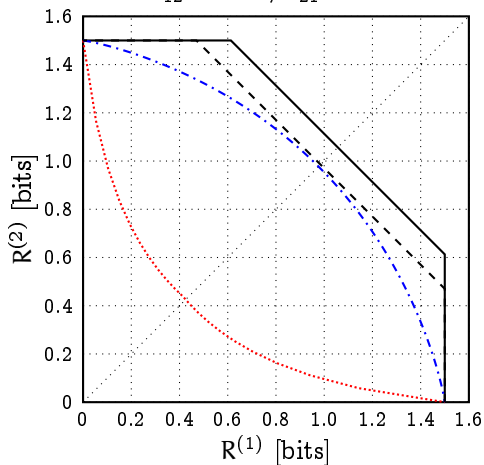
$a_{12} = 0.55, a_{21} = 0.45$



$a_{12} = 0.85, a_{21} = 0.75$



$a_{12} = 1.15, a_{21} = 1.05$



$a_{12} = 2.15, a_{21} = 2.05$

